Problem #26

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Summary: Is it true for non-orthogonal systems that decreasing redexes implies termination? If not, can some decent subclasses be delineated for which the implication does hold?

Let R be a term-rewriting or combinatory reduction system. Let "decreasing redexes" (DR) be the property that there is a map $\#$ from the set of redexes of R, to some well-founded linear order (or ordinal), satisfying:

- if in rewrite step $t \to_R t'$ redex r in t and redex r' in t' are such that r' is a descendant (or "residual") of r, then $\#r \geq \#r'$;

- if in rewrite step $t \to t'$ the redex r in t is reduced and r' in t' is "created" (t' is not the descendant of any redex in t), then $\#r > \#r'$.

Calling $\#r$ the "degree" of redex r, created redexes have a degree strictly less than the degree of the creator redex, while the degree of descendant redexes is not increased. The typical example is reduction in simply typed lambda calculus. In [Klo80] it is proved that for orthogonal term- rewriting systems and combinatory reduction systems, decreasing redexes implies termination (strong normalization). Does this implication also hold for non-orthogonal systems? If not, can some decent subclasses be delineated for which the implication does hold?

Comment sent by Vincent van Oostrom

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Contrary to what was claimed in [Klo80] (and in the statement of problem 26), decreasingness does not imply termination for orthogonal combinatory reduction systems. A counterexample can be found in Section 6.2.2 of the PhD thesis [Mel96], pp. 158-160.

The main application of the lemma, termination of rewrite systems having 'bounded production-depth', was recovered there ([Mel96], Theorem 6.5) in an axiomatic setting. For the case of higher-order rewriting this was shown in [Oos97].

Bibliography

[Klo80] Jan Willem Klop. *Combinatory Reduction Systems*, volume 127 of *Mathematical Centre Tracts*. Mathematisch Centrum, Amsterdam, 1980.

[Mel96] Paul-André Melliès. *Description Abstraite des Systèmes de Réécriture*. Thèse de doctorat, Université Paris VII, 20 Décembre 1996.

[Oos97] Vincent van Oostrom. Finite family developments. In Hubert Comon, editor, *8th International Conference on Rewriting Techniques and Applications*, volume 1232 of *Lecture Notes in Computer Science*, pages 308–322, Barcelona, Spain, June 1997. Springer-Verlag.

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