

Problem #53

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Date: June 1993

Summary: Are there hyper-recurrent combinators?

A term M in Combinatory Logic or λ -calculus is *recurrent* if $N \rightarrow^* M$ whenever $N \leftrightarrow^* M$ (this notion is due to M. Venturini-Zilli.) Let's call M *hyper-recurrent* if N is recurrent for all $N \leftrightarrow^* M$. (Equivalently, M is hyper-recurrent if $P \rightarrow^* Q \rightarrow^* P$ whenever $P \leftrightarrow^* Q \leftrightarrow^* M$.) Are there any hyper-recurrent combinators? (The problem comes up immediately when the Ershov-Visser theory [Vis80] for \leftrightarrow^* is applied to \rightarrow^* . It is known that hyper-recurrent combinators don't exist for Combinatory Logic [Sta91].)

Bibliography

- [Sta91] Richard Statman. There is no hyperrecurrent S,K combinator. Research Report 91-133, Department of Mathematics, Carnegie Mellon University, Pittsburgh, PA, 1991.
- [Vis80] A. Visser. Numerations, lambda calculus, and arithmetic. In Hindley and Seldin, editors, *Essays on Combinatory Logic, Lambda-Calculus, and Formalism*, pages 259–284. Academic Press, 1980.