Problem #79

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Summary: Does a system that is nonoverlapping under unification with infinite terms have unique normal forms?

Does a system that is nonoverlapping under unification with infinite terms (unification without "occur-check" [MR84]) have unique normal forms? This conjecture was originally proposed in [OO89] with an incomplete proof, as an extension of the result on strongly nonoverlapping systems [Klo80][Che81]. Related results appear in [OO93][TO94][MO94], but the original conjecture is still open. This is related to Problem 58. This problem is also related with modularity of confluence of systems sharing constructors, see [Ohl94].

Remark

The answer is yes if the system is also nonduplicating [Ver96]. A direct technique is given in [Ver96]. The nonduplicating condition can be relaxed under a certain technical condition [Ver96]. Some extensions to handle root overlaps are given in [Ver97] and a restricted version of the result in [Che81] is also proved using the direct technique in [Ver97].

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