On a Theory of Media Processing Systems Behavior, with Applications

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Abstract
In this article we provide a model for the dynamic behavior of media processing chains of tasks communicating via bounded buffers. The aim is to find the overall behavior of a chain from which performance parameters (such as start time and response time of individual tasks, chain end-to-end response time, number of context switches and resource utilization) follow. This behavior is characterized in terms of the elementary actions of the tasks that make up the chain. From this we derive corollaries that give design guidelines, supporting the goal of improving chain end-to-end response time and the optimization of resources, which is achieved by minimizing context-switching overhead and buffer sizes.

1. Introduction

1.1. Context

For the media processing in its Nexperia platforms, Philips has adopted a Pipes and filters architecture[1], known as TSSA. With this architecture, a media-processing system can be viewed as a graph in which nodes represent software components and edges represent buffers. Each component corresponds to a task, and the communication between tasks is buffered. Although hardware accelerators perform part of the media processing, the actual streaming is done in software on a single processor. To enable the construction of many complex systems in short time, Philips Research is investigating the composition of systems from reusable subsystems that share the platform resources. The resource sharing is based on the resource reservation paradigm [2] at the subsystem level [3]. Each subsystem represents a temporally independent sub-graph (the most common being a linear chain of components), and is provided with a guaranteed resource budget. Within each subsystem, the tasks are scheduled using fixed priority scheduling. Ideally, the behavior of each subsystem is dynamically controlled to ensure that the subsystem stays within its timing constraints ([4], [5]).

Due to the dependencies between the tasks, and their different behaviors, it is difficult to predict the behavior of the subsystem. Hence, it is difficult to determine resource budgets, to predict response times, to minimize buffer sizes and context switch overhead, and to reason about subsystem composition. The current practice lacks a theoretical underpinning that helps designers and integrators beyond intuition and experience, and is yet easy to understand. Such a theory is also needed to control the behavior at run time, to make sure timing requirements are met even in overload conditions.

1.2 Contribution and paper organization

Our research aims at providing an underlying theory that helps engineers to reason rigorously about system behavior and associated resource needs. It starts from the experimental observation that a media processing chain, assumes a repetitive behavior, the stable phase, after a finite initial phase. Starting from this observation we are building a theoretical model for the execution of streaming graphs in media processing systems. Our general strategy is to analyze streaming systems in an incremental manner starting from a simple theoretical case, to realistic streaming graphs that include branching and more complex types of components. This paper presents the initial case we analyzed, of a linear chain executing in a cooperative environment. Although this case misses a number of issues typical for media processing systems (such as variable computation times of tasks and timing constraints of the chain), it is essential for laying the foundation of our theory, for understanding what causes the repetitive behavior, and how this behavior can be controlled.

Our approach allows us to calculate the execution order of the components in a chain, expressed as a trace of actions [13] taken by each component. We formally prove that the behavior of the chain can be expressed as a unique trace, which assumes a repetitive pattern after a finite prefix. The trace is completely determined by the individual traces of the components, the topology of the chain, the capacities of the communication buffers, and the static priorities of the components. The unique trace of actions proves an excellent starting point for
further analysis. Simple additive formulas for the start times and response times of the individual tasks and the complete chain are immediately available. The number of context switches, and the position of the context switches in the component traces, which is an indicator for their overhead cost, can be extracted from the trace. Also given the individual traces of the components and the channel constraints (due to the buffered communication), we calculate the necessary and sufficient capacities for each buffer in the chain.

Our choice to formally express the behavior of the system in terms of traces comes from the fact that traces provide an adequate and simple way to specify the behavior of our streaming chains. Indeed other formal techniques such as Petri Nets [6] lead to very complex diagrams even when describing chains composed of quite simple components. CSP [13] presents another complication in the fact that buffers are considered processes (tasks) as well. In this case the tasks corresponding to components are assigned priorities while those corresponding to buffers do not. The complication comes from the fact that it is difficult to analyze the behavior of a system in which some tasks have priorities and others not.

This paper is organized as follows. In section 2 we discuss related work. Section 3 gives insight about basic notions of trace theory and sets the general approach for our work. Section 4 presents the system architecture we use and initial properties of the chain behaviors. Section 5 provides a characterization of the unique trace that expresses the chain behavior and answers to practical questions regarding the calculation and optimization of queue capacities, the length of the initial phase and the number of context switches. In section 6 we present timing aspects of the chain behavior and we show how the response time of the chain can be optimized. We end our presentation with conclusions in section 7.

2. Related work

Several attempts have been made to analyze message passing, streaming systems. Closely related work [7] considers also an execution model for video streaming chains inspired by TSSA. The article [7] presents an analysis method allowing the calculation of the worst-case response time of multiple video streaming chains based on the canonical form of the chains. The assumptions adopted are that tasks have fixed execution times, tasks are allowed to have equal priorities and the overhead introduced by context switches is ignored. Their approach is based on the response time analysis for tasks with deadlines beyond the periods [8].

Klein et al. [10] apply fixed-priority response time analysis to message-passing systems. The system is modeled in terms of events and event responses. Message handlers create new events if outgoing messages are sent at a different rate than incoming messages. Tasks are modeled as shared resources. The processing of a message by a task is modeled as an atomic action on the shared resource. This leads to the response-time analysis of a set of independent event responses with atomic access to shared resources.

Goddard [11] studies the real-time properties of PGM dataflow graphs, which closely resemble our media processing graphs. Given a periodic input and the dataflow attributes of the graph, exact node execution rates are determined for all nodes. This is very similar to the approach presented in [10]. The periodic tasks corresponding to each node are then scheduled using a preemptive EDF algorithm. For this implementation of the graph, the author shows how to bound the response time of the graph and the buffer requirements.

Both approaches consider complete task sets scheduled by a single scheduling algorithm, and are limited to task sets with deadlines equal to the period, i.e. without self-interference. Each of these attempts provided valuable insights, but none of them provided the theory that helps engineers to reason rigorously about system behavior and associated resource needs.

By focusing on the behavior of the chain, our work bears a close resemblance on work done by Gerber et al [12] on compiler support for real-time programs. One idea pursued in that work is to make sure that only relevant actions are performed between trigger and result-delivery, and that housekeeping tasks are delayed till after completion of the real-time part. In our approach, similar results are obtained by appropriate priority assignments to components.

3. Model

3.1. Components and traces

We use a simple intuitive syntax for the program text of the components using repetition (‘while’), selection (‘if’), sequential composition (‘;’) and basic statements (communication and computation actions). The set of basic statements of a component C is called its alphabet $\mathcal{A}(C)$. Alphabets of different components are disjoint.

The semantics is defined as the set of sequences that correspond to the possible execution sequences of these actions according to the program. These sequences are called the *traces* of the component. An example of a component with corresponding trace is $C: \{a; b; c; d\}$ (perform the four actions sequentially) with trace set $Tr(C) = \{(abcd)\}$.

A trace is a finite or infinite sequence of symbols. Traces $s$ and $t$ can be combined by concatenation to $st$;
if \( s \) is infinite this concatenation is just \( s \). Concatenation is generalized to sets of traces in the obvious way. The length of a trace \( t \) written as \(|t|\). For trace \( t \), \( \text{Prefix}(t) \) denotes the set of all prefixes of \( t \).

Besides this regular behaviour we introduce two extensions: infinite repetition and parallel composition. The infinite repetition corresponds to a set of infinite traces. For example,
\[
C: \text{while (true) do } \{a; b\} \text{ has traceset } \text{Tr}(C):\{(ab)^\omega\}.
\]

A realistic component example common to the video processing domain would be a sharpness enhancement component. Such a component receives as input a decoded video frame, performs a sharpness enhancement algorithm on this input, and finally sends as output the processed frame. This scenario is repeated infinitely many times. The syntax of the component is
\[
C: \text{while (true) do } \{\text{receive(frame); enhance(frame); send(frame)}\}.
\]
The traceset is similar as for the component above.

Parallel composition of two components is denoted by a parallel bar \( | \). The semantics for one processor is defined as the interleaving of the tracesets. For example \( C_0 | C_1 \), with \( C_0: \{a; b\} \) and \( C_1: \{c; d\} \) has traceset: \( \{(abcd), (achd), (acdb), (cadb), (cdab), (cabd)\} \).

We define the projection of a trace \( t \) on a certain alphabet \( A \), denoted by \( t \uparrow A \), as the trace obtained from \( t \) by removing all symbols not in \( A \) while maintaining the order given in \( t \). For trace \( t \) and symbol \( a \) we define the counting operator \( \# \) as follows:
\[
\#(t, a) = |t \uparrow \{a\} |
\]
Informally, \( \#(t, a) \) denotes the number of occurrences of \( a \) in \( t \).

### 3.2. States, invariants and channels

A traceset associated with a component represents all possible complete executions. A machine being in the middle of such an execution has only executed a prefix of such a trace. We associate such a prefix with a state during execution. Hence, the set of states during execution is characterized by the prefixes of the traceset, the prefix closure. For a traceset \( T \) we denote this set of states by \( \text{St}(T) \).

Properties that are true for all elements of a traceset are called invariants. For example, with
\[
C: \text{while (true) do } \{a; b\} \text{ we have}
\]
\[
I: 0 \leq \#(t, a) - \#(t, b) \leq 1 \text{ for all states } t \text{ in } \text{St}(\text{Tr}(C)).
\]
This invariant merely states that actions \( a \) and \( b \) alternate in every (partial) execution. In such an invariant we drop the trace argument from the function and simply say that \( 0 \leq \#a - \#b \leq 1 \) is an invariant of \( C \) (or of \( \text{Tr}(C) \)).

Invariant \( I \) above is an example of a synchronization condition. Actions \( a \) and \( b \) are in this example synchronized by virtue of the program syntax. We call such an invariant a topology invariant. However, instead of looking at invariants that the traceset already has, we can also impose invariants. This represents limitations on the execution of the atomic actions. These imposed invariants then lead to a restriction to the subset of traces and corresponding states for which the invariants hold.

As an example, consider once more the following:
\[
C_0 | C_1, \text{ with } C_0: \text{while (true) do } \{a; b\} \text{ and } C_1: \text{while (true) do } \{c; d\}.
\]
\[
\text{Tr}(C_0 | C_1) = \{s: s \text{ consists of elements of } \mathcal{A}(C_0) \cup \mathcal{A}(C_1) \wedge s \uparrow T \mathcal{A}(C_0) \in \text{Tr}(C_0) \wedge s \uparrow T \mathcal{A}(C_1) \in \text{Tr}(C_1)\}.
\]

We now decide that action \( a \) represents a send action and \( d \) a receive action on an unbounded channel. This interpretation leads to imposing the invariant \( 0 \leq \#a - \#d \).

This means that in the above set certain traces are ruled out as a behaviour. (Notice that the invariant must hold for all states derived from this set). Alternatively, we may decide that \( a \) and \( d \) form a synchronous channel as in CSP. This is captured in the invariant that \( a \) and \( d \) always follow each other immediately in the trace while their order is unimportant. This limits the traceset again; notice that this effectively orders \( c \) and \( b \) as well. As another example, assume that we assign \( C_0 \) a higher priority than \( C_1 \). This translates into the invariant that no \( C_1 \) actions can precede \( C_0 \) actions.

Send and receive actions via a channel \( c \) are denoted as \( c! \) and \( c? \) respectively like in CSP [13]. However, in contrast to CSP we use bounded channels by imposing the invariant
\[
(1) \quad 0 \leq \#c! - \#c? \leq \text{Cap}(c),
\]
with \( \text{Cap}(c) > 0 \) being the channel capacity.

We denote the number of elements in a channel \( c \) with \( L(c) \). We note that \( L(c) = \#c! - \#c? \). Therefore (1) can be rewritten as
\[
0 \leq L(c) \leq \text{Cap}(c).
\]

The introduction of constraints on the interleaved behaviour also leads to the notion of blocking. Consider a constrained traceset \( T \) and a particular state \( s \) of the system, i.e., an element of the prefix closure \( \text{St}(T) \). Suppose also that the traceset is the result of a parallel composition \( C_0 | C_1 \) with \( \mathcal{A}(C_0) \). Any action \( a \) in \( \mathcal{A}(C_0) \) such that \( sa \uparrow T \mathcal{A}(C_0) \) is a state of \( \text{Tr}(C_0) \) and \( sa \) a state of \( T \) is called a ready action of \( C_0 \) in state \( s \). If \( sa \) is not a state of \( T \) it is apparently not possible to execute \( a \) in the constrained set. We say that \( C_0 \) is blocked at \( a \) in state \( s \) of \( T \), denoted as \( C_0 \text{ b } a \) in \( s \) of \( T \) \). In most cases both \( s \) and \( T \) are clear from the context and then we leave them out. When \( T \) is given, then in any state \( s \) we can divide the set of components into blocked components (\( B(s) \)) and ready-to-run components (\( RR(s) \)).
We close this section with adding a few more concepts relevant to our model. We define Comp as a function taking as argument an action and returning the component with the alphabet to which action \( a \) belongs. Hence, for an action \( a \in \mathcal{A}(C) \), \( \text{Comp}(a) = C \).

Furthermore, consider a trace \( t \) written as \( t = s_0 \ a \ b \ s_1 \). If \( \text{Comp}(a) \neq \text{Comp}(b) \), then we say that a context switch occurs between \( \text{Comp}(a) \) and \( \text{Comp}(b) \) in state \( s_0 \ a \) of \( t \).

Finally, we define the number of context switches (NCS) function taking as argument a finite trace from a traceset \( T \), and returning the number of context switches occurring in the trace:

\[
NCS : T \rightarrow \mathbb{N}, \ NCS(\varepsilon) = 0, \ NCS(a) = 0,
\]

\[
NCS(abt) = \begin{cases} 
NCS(bt) & \text{if } \text{Comp}(a) = \text{Comp}(b) \\
NCS(bt) + 1 & \text{otherwise.}
\end{cases}
\]

3.3. Approach

The model presented above is a fairly conventional interleaving model [13]. Compared to regular trace semantics there are two major differences. First, we use the complete executions according to the syntax as the semantics, rather than the prefix closure. Secondly, we do not introduce any synchronization concepts in the syntax or the trace semantics. Instead we introduce the interpretation of atomic actions as well as execution policies by limiting attention to those traces that satisfy the interpretation. This makes the semantics simpler and the manipulation easier.

We consider a system consisting of a parallel composition of communicating components. The corresponding traceset is limited to those traces that satisfy the channel properties (1) for all channels. We call this the channel-consistent traces. On top of this set we impose priorities. This results in a single trace for the system, depending on the priority assignment. Characterizing precisely this trace is one of our targets. Besides this we add timing properties, which we can analyze as well. In short, we can analyze the system in terms of time and behaviour as a function of:

- the choice of the atomic action order in the components;
- the channel properties (e.g., the capacity);
- the priority assignment;
- the timing assignment.

4. A Streaming Pipeline

4.1. System architecture

We focus on systems consisting of a collection of communicating components connected in a pipelined fashion (Pipes and Filters architecture style). An instance of this architecture style, the TriMedia Streaming Software Architecture (TSSA) provides a framework for the development of real time audio-video streaming systems executing on a single TriMedia processor. A media processing system is described as a graph in which the nodes are software components that process data, and the edges are channels (finite FIFO queues) that transport the data stream in packets from one component to the next. A simple example of such a chain is presented in Figure 1.

![Figure 1. Chain of components.](image1)

Every connection between two components is implemented by two queues. One queue (forward queue) carries full packets containing the data to be sent from one component to the next, while the second queue (backward queue) returns empty packets to the sender component to recycle packet memory. The empty packets are returned to signal that the data has been received properly and that the associated memory may be reused. The capacity of \(fq_i\) is equal to the capacity of \( eq_i\). The system executes in a cooperative environment meaning that the environment will always provide input and always accept output. That means that blocking on the queues \(fq_0, eq_0, fq_N, eq_N\) is not possible. The initial situation of the chain is that all forward queues (except \(fq_0\)) are empty and that all backward queues (except \( eq_N\)) are filled to their full capacity. This is expressed in the following property:

\[
L(fq_i) = 0 \land L(eq_i) = \text{Cap}(eq_i) \land i, 0 < i \leq N.
\]

The behavior of \(Ci, 1 \leq i \leq N\), is the following (Figure 2): the component receives 1 full packet \( p \) from the input forward queue \( (fq_{i-1})\)\(\circ\), then receives 1 empty packet \( q \) from the input backward queue \( (eq_{i-1})\)\(\circ\), performs the processing \( (Ci)\)\(\circ\), recycles the input packet \( p \) from \(fq_{i-1}\) by sending it in the output backward queue \( (eq_i)\)\(\circ\) and finally, the result of processing \( q \) is sent in the output forward queue \( (fq_i)\)\(\circ\).

![Figure 2. A basic streaming component.](image2)
The syntax for the execution of a component \( C_i \) and the corresponding traceset are:

\[
\text{C}_i\text{ while (true) do}
\]

\{ receive(f_{qi-1}, p); receive(eq_i, q);
\]

\text{process}_fct_{ci}(p, q);

send(eq_i-1, 1); send(f_{qi}, 1); \},

with traceset \( Tr(C_i) \) = \{ (f_{qi-1}, eq_i, ci, eq_i-1, f_{qi}) \}.

for \( 1 \leq i \leq N \). Note that we write the atomic actions differently in the program syntax than in the trace in order to make the distinction clear.

We denote \( \text{set obtained by imposing \( Scc(t) \: t = \text{Tr}((\text{Comp} (a_i)) \cup A(C_i)) \cap Tr(C_i) \)) \) the corresponding traceset are:

\[
\text{St}(T_{cc})
\]

\[
\text{for an action} a \text{ and call them } \text{components (} A = \bigcup_{i=1..N} A(C_i) \text{), and } A^\omega \text{ the set of all infinite traces which are formed from actions in the set } A. \text{ The set of traces that results from the interleaving of the components is called } T_{ii}:
\]

\[
T_{ii} = \text{Tr}( \bigparallel_{i=1..N} \text{C}_i )
\]

In the following we restrict ourselves to the channel-consistent traces of this system, i.e. those traces in \( T_{ii} \) that for all their prefixes satisfy \( (1) \) for all channels in the system. Predicate \( Scc \) specifies this constraint. The set obtained by imposing \( Scc \) on all traces from \( T_{ii} \) is called \( T_{cc} \):

\[
Scc(t): (\forall s \in \text{Pref}(t): 0 \leq \#(s, f_{qi}) - \#(s, eq_i) \leq \text{Cap}(f_{qi}) \land 0 \leq \#(s, eq_i) - \#(s, eq_i) \leq \text{Cap}(eq_i))
\]

\[
T_{cc} = \{ t \in T_{ii} | Scc(t) \}
\]

Imposing \( Scc \) limits the order in which actions can interleave, and introduces blocking. For the new constrained set \( T_{cc} \) we recapitulate the definitions for ready-to-run and blocked components. Given \( s \) from \( St(T_{cc}) \), a component \( C_i \) is ready-to-run in state \( s \) when for an action \( a \in A(C_i) \) such that \( sa \uparrow A(C_i) \) is a state of \( Tr(C_i) \) we have that \( sa \in St(T_{cc}) \). Also, \( a \) is called a ready action of \( C_i \) in state \( s \). If \( sa \) is not an element of \( St(T_{cc}) \) it is not possible to execute \( a \) in the constrained set in which case we say that \( C_i \) is blocked at \( a \) in state \( s \) of \( T_{cc} \).

**Property 1.** For component \( C_i (1 \leq i \leq N) \), blocking at \( eq_i \) is not possible.

**Proof.** We prove this by contraposition by considering a state \( s \) where \( C_i \) would be blocked at \( eq_i \) and showing this state cannot exist. To be blocked at \( eq_i \), execution of \( C_i \) has to proceed until just before \( eq_i \). This means that for this state \( s \) we have

\[
x_i = 1
\]

In addition, blocking is caused by the channel consistency, hence

\[
(5) \# eq_i - \# eq_i = \text{Cap}(eq_i)
\]

We compare the situation between \( C_i \) and \( C_{i-1} \)

\[
x_i + y_i = 0
\]

\[
(4) y_i = 1
\]

The last term is at most 0 according to (1), which is a contradiction.

By symmetric reasoning we also have:

**Property 2.** For component \( C_i \), blocking at \( f_{qi} \) is not possible.

From these two properties we conclude that when blocking of components due to communication occurs, it is possible only at input actions.

**4.3. Precedence order constraints**

Next we introduce an additional restriction on the traces of \( T_{cc} \) in the form of a priority assignment. We define priority as a function \( P \) that returns for each component a unique natural number with the interpretation that a higher number means a higher priority. This translates into the invariant that in each state of \( T_{cc} \) where there are multiple components ready, an action of the one with the highest priority is selected. The imposed invariant is then specified as follows:

\[
\text{Spc}(t): (\forall s \in \text{Pref}(t): a_i \in A(C_i): t = sa \uparrow P(C_i))
\]

Limiting \( T_{cc} \) according to \( \text{Spc} \) gives \( T_{pc} \), the priority consistent traces:

\[
T_{pc} = \{ t \in T_{cc} | \text{Spc}(t) \}
\]

**Property 3.** \( T_{pc} \) has precisely one element.

**Proof.** We wish to prove that \( | T_{pc} | = 1 \).
In the first part of the proof we show that $T_{pc}$ consists of at least one element by construction according to the definition of $Scp$, following the algorithm below:

$$\rho_0 := e;$$

$$\rho_{n+1} := \begin{cases} \rho_n a_i & \text{if } \exists a: P(Comp(a)) = \max_{C \in RR(\rho_n)} P(C) \\ \rho_n, & \text{otherwise} \end{cases}$$

We consider $\rho = \lim_{n \to \infty} \rho_n$.

$Scp(\rho)$ holds for all $i$, and each $\rho_i$ is a prefix of $\rho$, hence $Scp(\rho)$ holds as well. Therefore $|T_{pc}| \geq 1$.

In the second part of the proof we show by contraposition that $|T_{pc}| \leq 1$.

We assume:

- $(\exists t_1, t_2 \in T_{pc}, t_1 \neq t_2)$
- $(\forall s, a_1, u_1: t_1 = sa_1u_1 \land a_1 \in A(C))$
- $P(Comp(a_1)) = \max_{C \in RR(\rho)} P(C))$
- $(\forall s, a_1, u_1, t_2 = sa_1u_1 \land a_1 \in A(C))$
- $P(Comp(a_1)) = \max_{C \in RR(\rho)} P(C))$

Consider the first state $s$ where the next action of the two traces differ, resp. $a_1$ and $a_2$. The fact that the priorities assigned to all components are unique and given that $P(Comp(a_1)) = P(Comp(a_2))$ implies that $Comp(a_1) = Comp(a_2)$. This implies that in state $s$ there do exist two ready actions of the same component, which is impossible according to the definition of ready action.

We denote the unique trace in $T_{pc}$ with $\rho$ and we are interested in the occurrence of context switches in $\rho$. Consider a context switch between components $C_i$ and $C_j$, i.e., $\rho = sa_1b_1u_1$ with $s \in A(\rho$, $a_1 \in A(C)$ and $b_1 \in A(C)$.

If $P(C) < P(Cj)$ we say that $C_j$ preempts $C_i$, we call this situation preemption and the context switch occurs due to preemption. In the other case we call this a context switch due to blocking.

5. Characterization of the unique trace $\rho$

The next goal is to characterize the single trace $\rho$ as a function of the priority assignment. In the first sub-section we present two lemmas and a theorem that allow the calculation of the trace at design time. In the second sub-section we show how the theorem helps answer practical questions about the minimum sufficient amount of memory, calculating and optimizing the length of the initial phase in the trace and minimizing the number of context switches.

5.1. Stable phase characterization

Consider a component $C_i$ that is a left local minimum in terms of priority ($\forall j: j < i$: $P(C_j) > P(C_i)$). Whenever it is executing the components to the left of it are blocked. However, the blocking can only be at a single place, according to the following lemma.

**Lemma 4.** Let $C_i$ be such that $(\forall j: j < i$: $P(C_j) > P(C_i)$) and consider a state $s$ in $St(\rho)$ such that the next action after $s$ in $\rho$ is one of $A(C)$. Then $C_j b eq_j^? \in s$ of $T_{pc}$ for all $j < i$.

In words, the lemma states that whenever actions of this local minimum component are executed, the components to the left of it are blocked at reading an empty packet from their right neighbors. The lemma specifies $C_j b eq_j^?$ in $s$ of $T_{pc}$ in order to clarify the blocking context given by $T_{pc}$. Indeed the blocking is due to the fact that predicate $Scc(t)$ where $t = s eq_j^?$ does not hold.

**Proof.** We prove this by contraposition again. Let $C_j$ be the leftmost component that is blocked elsewhere. Properties 1 and 2 already showed that blocking at output is not possible. The only possible actions where blocking due to channel communication or is possible is either at action $fq_j^!$ or $eq_j^?$. For $C_i$, blocking at $fq_j^!$ is not possible because of the cooperative environment hence, $eq_j^?$ is the only possible place to block. Therefore we must assume, $j > 1$ and $C_j b eq_j^?$. This implies that $y_j^1 = 0$, $x_j^0 = 0$, $\#fq_j^! - \#eq_j^?= 0$, and $\#eq_j^? - \#eq_j^? = Cap(eq_j^?)$.

From this follows:

0 =

$x_j^1 + y_j^1$ =

{ (2), (3) } # $fq_j^! - \#eq_j^? + \#eq_j^? - \#fq_j^!$

= { $\#eq_j^! - \#eq_j^?$ = 0, and $\#eq_j^? = Cap(eq_j^?)$ }

- $Cap(eq_j^?)$.

This is in contradiction with $Cap(eq_j^?)$ being strictly positive.

We have a symmetric lemma for $C_i$ as local right-minimum in a chain ($\forall j: j < i$: $P(C_j) > P(C_i)$).

**Lemma 5.** Let $C_i$ be such that $(\forall j: j > i$: $P(C_j) > P(C_i)$) and consider a state $s$ in $St(\rho)$ such that the next action after $s$ in $\rho$ is one of $A(C)$. Then $C_j b fq_j^!$ in $s$ of $T_{pc}$ for all $j > i$.

**Proof.** Analogous to proof of Lemma 4.

Lemmas 4 and 5 form the key for understanding the behavior of the whole pipeline. Consider component $C_m$ with minimal priority in the entire chain...
(P(C_m) = \min_{i=1}^{N} P(C_i)). The behavior of C_m is given by

\( (fq_m-?, eq_m?, cm, eq_m!, fqm!) \); Lemma 4 gives us that before the first execution of \( fqm_! ? \), all components C_i to the left of C_m have become blocked at eq_? and all components to the right of C_m are blocked at fqm_? . The unique trace recording the interleaved execution of components up to the first execution of an action of C_m is composed of \( 5 \sum_{j=1}^{m-1} \sum_{i=j} \text{Cap}(eq_i) + (m-1) \) actions (see Property 10 below), executed in an order determined by the priority of the components left of C_m. We denote this trace by \( t_{\text{init}} \) and we call it the initial phase in the execution of the system.

After \( t_{\text{init}} \), a few actions of C_m occur until \( eq_m! ? \). Immediately after that component C_m-1 runs; actually, all components C_j to the left of C_m will execute an iteration and return to being blocked at eq_?. The order of these actions is again entirely determined by the priority assignment and is of length \( 5(m-1) \). We call this trace \( t_2 \) and we denote the set of components to the left of C_m with \( S_2 \). Next, C_m outputs: \( fqm! \), which results in the enabling of the components to the right of C_m that execute a full iteration until each component is blocked at the same place again. This trace has length \( 5(N-m) \) and we call it \( t_3 \) (we denote the set of components to the right of C_m with \( S_3 \)). At this point C_m is again the only component ready-to-run and the situation described above repeats. The order of actions in the traces \( t_i \) and \( t_k \) is always the same because the priority assignment does not allow any deviation. In short this means that the system follows a repetitive behavior. We arrive at the following theorem.

Theorem 6 (Stable Phase Theorem). The pipeline system assumes a repetitive behavior after a finite initial phase. The complete behavior is characterized by

\[ \rho = t_{\text{init}} (fq_m_? \eq_m_? \cm \eq_m_! \tl \fqm_! \tr)^n \]

We denote the trace \( (fq_m_? \eq_m_? \cm \eq_m_! \tl \fqm_! \tr)^n \) with \( t_{\text{stable}} \) and we call it the stable phase in the execution of the system.

Corollary 7. The stable phase starts when C_m executes for the first time.

5.2. Practical applications

5.2.1. Channel usage, minimizing channel capacities

The following corollary shows what the number of packets is in all queues at the beginning of any iteration of \( t_{\text{stable}} \), more precisely right after C_m executes \( fqm_! ? \). This information becomes useful later when we address issues related to the chain response time for each packet in the chain (Section 6).

Corollary 8. Given a state \( s \) in \( St(\rho) \) such that

\[ s = t_{\text{init}} (fq_m_? \eq_m_? \cm \eq_m_! \tl \fqm_! \tr)^n \]

the following statements hold:

a. \( \forall i: 1 \leq i < m \): \( L(fqi) = \text{Cap}(fqi) - 1 \wedge L(eq_i) = 0 \)

b. \( \forall i: m \leq i < N \): \( L(fqi) = 0 \wedge L(eq_i) = \text{Cap}(eq_i) \)

Proof. a. When C_m executes \( fqm_! ? \), components C_i (1 \leq i \leq m) are blocked at eq_?. This implies that \( L(eq_?) = 0 \). From here follows that there are \( \text{Cap}(fqi) \) full packets available for consumption. Given the order of actions in the individual traces of components, each C_i executes first \( fqi_? \) before eq_?, meaning that when C_m becomes blocked at eq_? it will have already executed \( fqi_? \), which makes that each \( fqi \) contains \( \text{Cap}(fqi) \) full packets and one packet is inside the component C_i.

b. Lemma 5 implies that when C_m executes \( fqm_! ? \), components C_i (\( m < i < N \)) are blocked at action fqi_?. This implies that \( L(eq_?) = 0 \). From here follows that there are \( \text{Cap}(eq_?) \) full packets available for consumption. Given the order of actions in the individual traces of components, each C_i executes first \( eq_? \) before fqi_?.

Corollary 9 shows the minimum necessary and sufficient capacity of the queues that interconnect the components. It is relevant to know the minimum amount of memory that the system needs for communication between components.

Corollary 9. The minimum queue capacity necessary and sufficient for any queue \( fqi \) (and implicitly eq_i) \( 1 \leq i < N \), is 1.

Proof. Follows directly from Corollary 8.

5.2.2. Optimizing the initial phase length

In this subsection we calculate the length of the initial phase (Property 10) and show how \( t_{\text{init}} \) can be reduced and even eliminated (Corollary 11). Reducing the initial phase is important because designers assign budgets to the chain based on the resource requirements of the stable phase.

Property 10. The length of the initial phase in number of actions is

\[ 5 \sum_{i=1}^{m-1} \sum_{j=1} \text{Cap}(eq_j) + (m-1). \]

Proof. Due to lack of space we give a sketched proof below, the full proof is available on demand. The number of actions executed until the execution of the system reaches the stable phase is the number of actions executed by all C_i (\( 1 \leq i < m \)) until all their backward input queues (eq_i, \( 1 \leq i < m \)) are drained. All these queues are drained after \( 5 \sum_{i=1}^{m-1} \sum_{j=1} \text{Cap}(eq_j) \) actions.
After that, all \( m-1 \) components \( C_i \) \((1 \leq i < m)\) will execute action \( f_{q_i} \) and then they become blocked at action \( eq_i \) \((1 \leq i < m)\). That leaves us with the total number of actions for the initial phase:

\[
|t_{in}| = \sum_{i=1}^{m} \sum_{j=1}^{1} \text{Cap}(eq_j) + (m-1).
\]

**Corollary 11.** \( |t_{in}| \) can be reduced by decreasing \( m \) or decreasing the capacity of the queues to the left of \( C_m \) in the chain.

**Proof.** Direct consequence of Property 10.

5.2.3. Minimizing the number of context switches

In this subsection we show that the priority assignment influences the number of context switches in \( \rho \) and we give insight on what is the appropriate priority assignment that minimizes \( \text{NCS}(t_{in}) \) and \( \text{NCS}(t_{stable}) \).

**Theorem 12.**

a. Minimum \( \text{NCS}(t_{in}) \) is achieved when

\[
P(C_i) = \min_{i=1..N} P(C_i).
\]

b. The minimum NCS during one iteration of \( t_{stable} \) can be achieved either when:

i. \( P(C_i) = \min_{i=1..N} P(C_i) \).

or when

ii. \( P(C_i) < P(C_{i+1}) < \ldots < P(C_{N-1}) \) with \( \forall \, i: 1 \leq i < N-1, \text{Cap}(f_{q_i}) = 2 \).

**Proof.**

a. Corollary 11 implies that for \( m=1 \), the initial phase is eliminated, therefore \( \text{NCS}(t_{in}) \) is 0.

b. **Theorem 6** shows that regardless of the priority assignment, context switches occurring during the execution of components \( C_i \) \((i \neq m)\) due to blocking cannot be avoided; hence the only context switches that can be eliminated are the ones due to preemption. In the case of \( C_m \), this component does not block but is preempted after both actions \( eq_m \) and \( f_{q_m} \). Only one of these two context switches can be avoided by choosing \( m = 1 \) or \( m = N \).

In case \( i., m = 1 \). In this case, regardless of the priority assignment to the components \( C_i \) \((i > m)\), the plus of context switches due to preemption is already eliminated because preemption and blocking occur after the same action \( f_{q_i} \), therefore \( \text{NCS}(\rho) \) cannot be optimized more.

In case \( ii., m = N \). In this case, preemption and blocking occur after different actions. The priority assignment suggested implies that all context switches caused by preemptions are avoided in trace \( t_{st} \), each component executes until it is again blocked. Moreover, with this priority assignment, had the lengths of the queues been 1, additional context switches would have occurred due to the blocking on the input forward queues of components \( C_2,\ldots,C_{N-1} \) (Corollary 8) therefore the additional condition on the minimal length of queues must be imposed for case \( ii. \): \( \forall \, i: 1 \leq i < N-1, \text{Cap}(f_{q_i}) = 2 \).

**Tradeoff**

a. The priority assignment suggested by **Theorem 12 - ii** implies that all context switches caused by preemptions were eliminated, at the cost of an initial phase.

b. The priority assignment suggested by **Theorem 12 - i**. achieves a minimum \( \text{NCS}(t_{stable}) \) at the cost of more memory needed \( (\forall \, i: 1 \leq i < N-1, \text{Cap}(f_{q_i}) = 2) \).

6. Timing

In order to analyze the system from the perspective of time we extend our modeling to include time. Using this we explain how the chain response time can be optimized.

6.1 Theoretical concepts

We introduce function \( \delta : A \times N \times T_{st} \rightarrow N \) that for each occurrence of an action from alphabet \( A \) in a trace \( t \) from \( T_{st} \) returns the computation time needed to execute it. The computation time is expressed in CPU cycles. \( \delta(a, k, t) \) denotes the computation time of the \( k^{th} \) occurrence of action \( a \) in trace \( t \). Important to recognize is that the computation times of different occurrences of an action can be different. In the case of the media processing systems, the computation times of the actions are variable due to dependency on input. For the MPEG 2 streams the values of function \( \delta \) can be provided by applying techniques shown in [14].

We denote the \( k^{th} \) occurrence of an action \( a_i \) \( \in A(C_i) \) in a trace \( t \) with \( a_i^k \), and for \( \delta(a, k, t) \) we use the short hand notation \( \delta(a_i^k) \) when \( t \) is clear from the context. The \( k^{th} \) occurrence of \( a_i \) processes the \( k^{th} \) packet.

We introduce the schedule function \( \sigma : A \times N \times T_{st} \rightarrow N \) of the concurrent execution of components \( C_i \), \( i=1..N \) on a processor. The schedule function returns the start time for each action occurrence. For \( \sigma(a_i, k, t) \) the short hand notation we use is \( \sigma(a_i^k) \). The finish time of \( a_i^k \) is \( \sigma(a_i^k) + \delta(a_i^k) \).

In general, a valid schedule \( \sigma \) for a trace \( t \) in trace set \( T_{st} \) satisfies the following criterion:

\[
\text{For each } t = s_1...a_i^k...s_2, \sigma(b)^2 \geq \sigma(a_i^k) + \delta(a_i^k).
\]

In reality, when a chain executes in a cooperative environment, components execute the soonest possible given the channel and priority constraints. This leads to the schedule satisfying the following:
For each $t = s_j a_k^i b_j^i s_l$,

\[
\sigma(b_j^i) = \sigma(a_k^i) + \delta(a_k^i), \quad \text{if } i = j,
\]

\[
\sigma(b_j^i) = \sigma(a_k^i) + \delta(a_k^i) + \Delta_c(a_k^i, b_j^i, s_j), \quad \text{otherwise.}
\]

$\Delta_c(a_k^i, b_j^i, s_j)$ is the overhead introduced by a context switch between action occurrences $a_k^i$ and $b_j^i$ executing after prefix $s_j$ of trace $t$. The overhead introduced by each context switch due to instruction cache misses can be calculated at design time. That is because Theorem 6 allows us to calculate at design time the position of context switches in trace $\rho$. This allows the calculation at design time of the overhead introduced by a context switch due to the replacing of instructions in the instruction cache. The overhead introduced by data cache misses depends on the input data; it can be predicted using techniques as in [14].

We define the response time $R_{\text{ck}}$ (Figure 3) of component $C_i$ ($1 \leq i \leq N$) that processes the $k^{th}$ packet:

\[
R_{\text{ck}} = \sigma(f_{q0}^i) + \delta(f_{q0}^i) - \sigma(f_{q0}^i, \rho), \quad \text{for all } 1 \leq i \leq N.
\]

\[\begin{array}{c}
\text{CPU cycles} \\
\sigma(c_i^k) + \delta(c_i^k) \\
\text{RTC}_k \\
f_{q0}^k \\
f_{qN}^k \end{array}\]

$\text{NA}_k=11$

\[\begin{array}{c}
f_{q0}^k \\
f_{qN}^k \end{array}\]

Figure 3. A schedule associated with a trace. Chain response time corresponding to $\text{NA}_k$

Additionally we define the response time of the chain (RTC) for packet $k$ as the time counted from the moment that the packet starts being processed by the first action of $C_1$ until the finish time of the last action of $C_N$:

\[\text{RTC}_k = \sigma(f_{qN}^k) + \delta(f_{qN}^k) - \sigma(f_{q0}^k, \rho), \quad \text{for all } 1 \leq k \leq N.\]

We denote the number of actions occurring from the arrival of packet $k$ (as a result of action $f_{q0}^k$) until the packet leaves the chain (as a result of $f_{qN}^k$) with $\text{NA}_k$. For instance in Figure 3, $\text{NA}_k$ is 11.

Knowing trace $\rho$ allows the calculation of the number of context switches (NCS ($\rho$)) and $\text{NA}_k$ for any packet $k$ in the input stream. Trace $\rho$, the function $\delta$, and the cost of each context switch $\Delta_c$, allows the calculation of the schedule function. From here, the start and finish time of each action in $\rho$, the response time for each component, the RTC, for each $k$ packet, the CPU and memory utilization can be also be calculated.

### 6.2 Optimizing chain response time

The response time of the chain for a packet $k$ can be optimized when optimizing $\text{NA}_k$. We show in the following that $\text{NA}_k$ is minimum when $P(C_i) = \min_{i=1..N} P(C_i)$.

\[\begin{array}{c}
P(C_i) \\
P(C_j) \end{array}\]

Figure 4. Priority assignment and queue occupation at the beginning of a $t_{\text{stable}}$ iteration.

Consider the example depicted in Figure 4-a) and b). The two figures show two possible priority assignments for a chain consisting of three components. We recall that similar as before we denote with $C_i$ the component with minimum priority in the entire chain $P(C_i)=\min_{i=1..N} P(C_i)$. We consider $\text{Cap}(f_{q0}) = 2$ for all $1 \leq i < N$. After calculating trace $\rho$ for the given priority assignments, we find that in case a) (when $m>1$), for packets $k = 1..3$ that enter the chain during the initial phase: $\text{NA}_1 = 26$, $\text{NA}_2 = 36$, $\text{NA}_3 = 31$. For packets $k>3$ that enter the chain during the stable phase $\text{NA}_k = 52$.

In case b) (when $m=1$), $\text{NA}_k = 15$, for all $k \in \mathbb{N}$, $k>0$. The reason for $\text{NA}_k$ being lower in case b) than in case a) is that in case a) packets have to “wait” for the end of the initial phase before being processed to the end of the chain, while in case b) the initial phase does not exist. In addition, in b), all packets are processed to the end of the chain immediately when they enter the chain, while in a) even during the stable phase, after entering the chain, packets must first wait for other packets preceding them in the FIFO queues in front of $C_m$ to be processed.

We generalize these observations in the following theorem.

**Theorem 13.** $\text{NA}_k$ is minimal for all $k \in \mathbb{N}$, $k>0$ when $P(C_i) = \min_{i=1..N} P(C_i)$.

**Proof.** We distinguish the case that $m>1$ from $m=1$.

**A.** If a $k^{th}$ packet has been inserted in the chain during the initial phase then,

\[1 \leq k \leq \sum_{i=1}^{m-1} \text{Cap}(f_{qi})+1 \quad \text{(Corollary 8)},\]

\[\begin{array}{c}
P(C_i) \\
P(C_j) \end{array}\]

Figure 4. Priority assignment and queue occupation at the beginning of a $t_{\text{stable}}$ iteration.
and $t_{\text{total}}$ can be written as: $t_{\text{total}} = u_1 f_{q_0} a_k u_2$. We denote $f_{q_0} a_k u_2$ with $v_k$. In this case $N_{A_k} = |w| + k * 5*N$.

Indeed, any packet $k$ inserted in the chain during the initial phase will not be processed to the end of the chain before the initial phase is completed. Therefore, regardless of which position in the trace $t_{\text{initial}}$, $f_{q_0} a_k$ occurs, $|w|$ actions must be executed before even the first packet in the stream can be processed to the end of the chain. The length of $|w|$ for any packet $k$ other than the first packet depends on the priority assignment. For $k=1$, $|w| = |t_{\text{initial}}|$.

Once the initialisation phase is completed, packet $k$ must "wait" for the other $k-1$ packets to be processed to the end of the chain. For each packet this happens within one iteration of the stable phase ($t_{\text{stable}}$), that is $5*N$ actions. That means that the total number of actions that occur between $f_{q_0} a_k$ and $f_{q_0} a_k$ is $N_{A_k} = |w| + k*5*N$ (we counted the iteration for processing packet $k$ itself too).

B. Any packet $k > \sum_{i=1}^{m-1} \text{Cap}(f_{q_i}) + 1$ arrives during the stable phase. Each packet $k$ arrives during the iteration of $t_{\text{stable}}$ that processes packet $k - \sum_{i=1}^{m-1} \text{Cap}(f_{q_i}) - 1$ to the end of the chain (more precisely during the trace $t_i$).

We express $t_i$ as $s_1 f_{q_0} a_k s_2$, and denote $f_{q_0} a_k s_2$ with $v_k$. The entire iteration can be written as:

$$f_{q_{m-1}} e_{q_{m-2}} e_{q_{m-1}} t_i f_{q_m} t_q = f_{q_{m-1}} e_{q_{m-2}} e_{q_{m-1}} s_1 f_{q_0} a_k s_2 f_{q_m} t_q = f_{q_{m-1}} e_{q_{m-2}} e_{q_{m-1}} s_1 v_k f_{q_m} t_q.$$  

This implies that $|v_k| + |t_q| + 1$ actions are executed from the arrival of packet $k$ (as a result of $f_{q_0} a_k$), until this iteration is completed. After these actions, packet $k$

\[ m-1 \]

must “wait” for $\sum_{i=1}^{m-1} \text{Cap}(f_{q_i})$ packets to be processed to the end of the chain. When counting also the iteration that processes packet $k$ itself, we find that

$$N_{A_k} = |v_k| + |t_q| + 1 + (\sum_{i=1}^{m-1} \text{Cap}(f_{q_i}) + 1)*5*N$$

$$= |v_k| + (N-m)*5*N + 1 + (\sum_{i=1}^{m-1} \text{Cap}(f_{q_i}) + 1)*5*N$$

\[ ii. m = 1. \]

When $P(C_i) = \min_{i=1..N} P(C_i)$, according to Corollary 11, the initial phase is eliminated completely, each packet $k$ arrives during the $k^{th}$ iteration of $t_{\text{stable}}$ and does not have to wait for other packets in front of it in the queues before it is processed. From here follows that

$$N_{A_k} = 5*N.$$

When comparing the formulas obtained for $N_{A_k}$ in cases $i.$ and $ii.$, it follows directly that for $m=1$, $N_{A_k}$ is minimal.

7. Conclusions

In this paper we have presented a model for the dynamic behavior of linear media processing chains executing in a cooperative environment. We express the behavior of the chain as a trace $\rho$ of the actions of the components that make up the chain. We have formally proven that the trace $\rho$ becomes repetitive (the stable phase) after a finite prefix (the initial phase) and we have shown that trace $\rho$ can be calculated at design time. This approach allows the calculation and optimization of the capacities of the queues between components, of the initial phase, of the number of context switches, and of the response time of individual components and the entire chain.

The repetitive nature of the chain is an important property that also makes reasoning about composition of chains much easier. Designers have only to reason in terms of patterns of execution at the level of the chain instead of reasoning about the individual behavior of components within the whole system. This approach also makes systems open in the sense that the effect of inserting (or withdrawing) components from a chain can be rigorously predicted and controlled.

Future work will tackle chains including components with periodic behavior and components whose behavior is influenced by the input stream. Future contributions will deal with variable computation times for tasks, and chains with timing requirements. We will also investigate other chain topologies such as branching chains and chains composition. Finally we plan to study the applicability of this approach on streaming systems executing on multiprocessor platforms.

8. References