GOSSIP AND THE COMPOSITION OF METRICS

JAN DRAISMA

I will discuss two related combinatorial theorems for which I do not know combinatorial proofs. The first theorem concerns pessimal gossiping: since the 70's it is known that n gossipers who start out with individual pieces of gossip that are exchanged via phone calls between pairs in which each gossiper tells the other all s/he knows, need at least $2n-4$ phone calls to achieve that everyone knows all gossips; and this bound is sharp for $n > 3$. This is optimal gossiping. *Pessimal* gossiping concerns the longest sequence(s) of phone calls such that in each call at least one of the parties learns something new. We prove that such a sequence cannot be longer than $\binom{n}{2}$. Again, this bound is sharp.

The second theorem concerns the structure of the monoid generated by all $n \times n$ distance matrices (i.e., symmetric matrices satisfying the triangle inequality) under min-plus matrix multiplication. The theorem says that this is a polyhedral complex of dimension $\binom{n}{2}$ (and not more). For small values of n we know the structure of this complex in more (combinatorial) detail.