CARTAN-BOTT PERIODICITY FOR COMPACT FORMS OF KAC-MOODY ALGEBRAS OF TYPE E_n

MAX HORN

Kac-Moody algebras generalize finite dimensional semisimple Lie algebras (such as sl_n) to the infinite dimensional, and are of great interest in theoretical physics. In particular, the Kac-Moody algebras (and groups) of type E_{10} pop up in string theory.

Given an irreducible simply-laced diagram Π, we sketch the constructing of a real Kac-Moody algebra $\mathfrak{g}$ of type Π, and of its compact form $\mathfrak{k}$ (the fix points of a Cartan-Chevalley involution). As an example, for $\Pi = A_n$ one has $\mathfrak{g} \cong \mathfrak{sl}_{n+1}(\mathbb{R})$ and $\mathfrak{k} \cong \mathfrak{so}_{n+1}(\mathbb{R})$. In the classic finite dimensional case, the Lie algebras $\mathfrak{k}$ are simple, or the sum of two simple subalgebras. In contrast, in the infinite dimensional case, $\mathfrak{k}$ is far from simple, and in fact admits finite-dimensional quotients.

We discuss which quotients occur in case $\Delta = E_n$ for $n \geq 3$. Interestingly, the quotients follow a nice periodic behavior, related to the classic principle of Cartan-Bott periodicity. Our proof of this boils down to a nice combinatorial argument.

This is joint work with Ralf Köhl.