Fast multiplication in polycyclic groups

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A group is called polycyclic if it has a finite subnormal series with cyclic factors. The investigation of polycyclic groups was started in the 1930s by Hirsch and was later continued by Baer, Mal'cev and Hall among others.
Every polycyclic group G can be finitely presented by a so called polycyclic presentation, which is a relatively easily manageable representation for a computer. One of its advantages is, that every element $g \in G$ has a normal form with respect to the chosen polycyclic presentation.
Finite solvable groups are polycyclic and algorithms for them were developed in the 1980s by Laue, Neubüser and Schoenwaelder. More recently computations with infinite polycyclic groups have been shown to be practical. The efficiency of calculations with polycyclic presentations depends on the ability to compute quickly the normal form of the product of two elements given in normal form. This process is called collection.
In this talk I want to give some examples that show that polycyclic groups are interesting objects to study. Further I want to describe an algorithm for fast collection in polycyclically presented groups, which uses the Mal'cev correspondence between radicable torsion-free nilpotent groups and nilpotent Lie algebras, and compare its performance with the classical strategy "Collection from the left".