On the existence of identifiable reparametrizations for linear compartment models

Abstract

Linear compartment models arise in many fields, such as systems biology and pharmacokinetics. These models describe the transport of a given material through a set of compartments, and can be represented by a directed graph. The parameters of a linear compartment model correspond to the transport rates between different compartments, and these are usually estimated by performing an experiment. But before setting up an experiment, one should consider wether the experimental data will correspond to a unique set of parameter values; in this case the model is called identifiable. In this thesis we consider generic local identifiability, meaning that a sufficiently general set of parameter values is at least locally a unique solution to the parameter estimation problem.

A problem that arises is what do to when a model is unidentifiable. A common approach is to search for an identifiable reparametrization of the model: a map which reduces the number of parameters, such that the reduced model is identifiable. Of particular interest in biological applications are rational scaling reparametrizations, which correspond to a rational scaling of the state variables.

We study a specific class of models, which are known to be unidentifiable. Earlier studies presented a criterion to decide whether a model has an identifiable scaling reparametrization, based on the dimension of the input-output map. Using algebraic geometry and graph theory, we translate this criterion to a criterion based on the rank of a bi-adjacency matrix. This new criterion can be evaluated by a randomized algorithm with asymptotic complexity of $O(n^6)$ operations. Furthermore, we present several new constructions to obtain a graph with an identifiable scaling reparametrization. Using these constructions, a large subclass of graphs which have an identifiable scaling reparametrization is obtained. This leads to a procedure of subdividing or deleting edges to ensure that a model has an identifiable scaling reparametrization.