On a Generalization of the GCD for Intervals in R^+

or

how can a camera see at least 1 tone for unknown T_{exp}

Author: Stan Baggen (stan.baggen@philips.com)
Philips Research Eindhoven

May 15, 2014

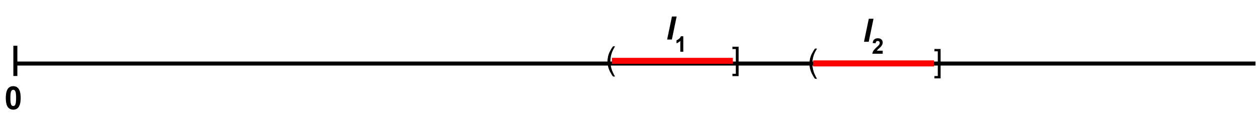
Abstract

We consider the situation where we want to transmit digital information from a luminaire to the camera of a smartphone or tablet using Visible Light Communication (VLC). In VLC, the light intensity of a transmitting luminaire is modulated, for instance by using Frequency Shift Keying (FSK).Typically, while recording a movie, each pixel of the camera records the average light intensity during T_{exp} seconds (the exposure time) immediately preceding the read-out. As a result, modulating frequencies $f=n/T_{exp}$ are invisible to the camera for all integer n. The camera uses a certain $T_{exp} \leq 1/30$, where the actual T_{exp} is unknown to the transmitter.

We asked ourselves the question if a transmitter can have two fixed frequencies f1 and f2, such that integer multiples of $1/T_{exp}$ cannot simultaneously be close to both f1 and f2, for any real $T_{exp} \leq 1/30$.

It turns out that a solution can be found by introducing the following generalization of the concept Greatest Common Divisor (GCD) for half-open intervals over R^+:

$$\mathrm{GCD}(I_1, I_2) := \max \left\{ a \in R \mid \exists_{n,m \in N} \left(na \in I_1 \wedge ma \in I_2 \right) \right\}$$



We discuss some interesting properties of this generalized GCD.