Three scheduling problems that have remained open for many years can be solved polynomially abstract

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During this presentation, Boolean Linear Programming (BLP) models are presented for three single machine scheduling problems with equal-length jobs and different release dates, and it is proven that they are polynomially solvable. The objective of the first problem, in which preemption is allowed, is to minimize the total weighted completion time. The objective of the other two problems is to minimize the total weighted tardiness. The second problem is preemptive, the third is not. To this date, the complexity status of these problems has remained open. The open complexity status was mentioned by Labetoulle et al. in 1984. See also the website of Drr,

`http://www.lix.polytechnique.fr/~durr/OpenProblems/1_rj_pmtn_pjp_sumWjCj/}`

The BLP models are based on the Assignment Problem formulation, which is a well-known polynomially solvable problem in combinatorial optimization. The Assignment Problem can be used to solve many problems, including for example the single machine scheduling problems in which the total weighted tardiness is minimized and in which all jobs have unit processing times and different release dates. In order to solve the three problems mentioned above only preoptimized permutations are incorporated into the Assignment Problem formulation. The proof that the problems are polynomially solvable is based on the notion of Total Dual Integrality.

Keywords: (Non-)preemptive scheduling; Equal-length jobs; Release dates; Weighted completion time; Weighted tardiness; Total dual integrality.