A tropical approach to secant varieties

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Tropical geometry turns questions about algebraic varieties into questions about polyhedral complexes. Though one might lose information in this process, the dimensions of certain secant varieties seem to behave well under tropicalisation.

Consider the following example: let d be a natural number, and consider the cube $K = \{0, 1\}^d$ in R^d. Let k be such that $k(d + 1) \leq 2^d$ (the "sphere packing bound" for binary codes of minimal distance 3). Are there always k affine-linear functions $f_1, ..., f_k$ on R^d such that the "winning set"

$$W_i := \{p \in K | f_i(p) < f_j(p)\ \forall j \neq i\}$$

of f_i spans R^d affinely, for every i ?

If there exists a binary code in K of minimal distance 3 and of size k, then the code-words can be turned into such f_i, for which W_i contains a Hamming ball of radius 1, and the answer is "yes". But the existence of such f_i is weaker than the existence of a code: for $d = 6$ there exist 9 such f_i, but the maximal size of a binary code of distance 3 is 8.

This "polyhedral question" comes up naturally when one tropicalises the problem of determining the higher secant varieties of the set of decomposable tensors in a the d-fold tensor power of C^2.

I will give a short introduction to secant varieties, tropical geometry, and the application of the latter to the former them that leads to such beautiful polyhedral-combinatorial problems.