Polarisation in Invariant Theory

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When a group G acts linearly on a vector space V over a field K, an invariant is a polynomial on V which is constant on G-orbits. If f is such an invariant, and if W is another vector space with G-action, then we may construct invariants on W by composing f with all possible G-equivariant polynomial maps $W \to V$.

If $V = M^q$ and $W = M^p$ (q-tuples and p-tuples, respectively) for a third G-module M, and if we only consider the natural G-equivariant linear maps $M^p - > M^q$ induced by linear maps $K^p - > K^q$, then the invariants on M^p thus constructed from an invariant f on M^q are called *polarisations* of f.

Sometimes all invariants on M^p can be expressed in polarisations of invariants on M^q for some $q < p$. Sometimes this is not true, but weaker variations on this statement still hold. I will give an overview of known results of this type.