On the classification of rank two commutative semifields

Michel Lavrauw

Universitat Politécnica de Catalunya, Barcelona, Spain

A *semifield* (also called division algebra) is an algebra satisfying the axioms for a skew field except (possibly) associativity. The study of finite semifields was initiated almost a century ago by L. E. Dickson and a good survey of the earlier work can be found in an article by D. E. Knuth (1965). Throughout this talk the term semifield will refer to a finite semifield. The relevance of semifields in the theory of translation planes is well known and can be found in Dembowski's book "Finite Geometries" (1968). Since their origin, semifields have been shown to be related to a large number of interesting geometric structures besides translation planes, such as flocks of a quadratic cone, spreads, ovoids and BLT-sets of polar spaces, translation generalized quadrangles and eggs in projective spaces; they play a key role in finite geometry. The *middle nucleus* of a semifield S is the set of elements x of S for which $a(xb) = (ax)b$ for all a, b in S, and it is straightforward to show that the middle nucleus of S is a finite field. The most interesting semifields from our point of view are those which are commutative and of rank two over their middle nucleus (*rank two commutative semifields*), because of the large number of related geometric objects. The classification of rank two commutative semifields of even characteristic was obtained by S. D. Cohen and M. J. Ganley in 1982; no "non-trivial" (i.e. not a field) rank two commutative semifields of even characteristic exist. In this talk we will consider rank two commutative semifields of odd characteristic in which case the classification is still incomplete.