Finite numbers of initial ideals in non-Noetherian polynomial rings

In my talk, we will study ideal chains in the polynomial ring $R = K[x_{i,j} \mid 1 \leq i \leq c, j \in \mathbb{N}]$ that are invariant under the action of the monoid $Inc(\mathbb{N})$ of strictly increasing functions on $\mathbb{N}$, which acts on R by shifting the second variable index. I will show that for every such ideal chain, the number of initial ideal chains with respect to term orders on R that are compatible with the action of $Inc(\mathbb{N})$ is finite. As a consequence of this, we will see that $Inc(\mathbb{N})$-invariant ideals of R have only finitely many initial ideals with respect to $Inc(\mathbb{N})$-compatible term orders. In the second part of my talk, I will give a full classification of the $Inc(\mathbb{N})$-compatible term orders on R for arbitrary c.