

Title: Locally finite polynomial maps

Abstract:

In my opinion, (invertible) polynomial maps are the natural generalization of (invertible) linear maps, in the sense that they can be applicable in many cases where people are using linear maps. Also, a lot of conjectures on polynomial automorphisms are generalizations of trivial facts for linear maps. For example, the *notorious* Jacobian Conjecture: “if F is a polynomial map, then

$$\det(\text{Jac}(F)) \in \mathbb{C}^* \Rightarrow F \text{ is invertible.}”$$

Although very easy to state, this ridiculous problem is still completely open (well, except dimension 1). And this shows the problem - the theoretic foundation of polynomial maps is too weak to let them be of much practical use for now. For example, in linear algebra one has the Cayley-Hamilton-theorem, which is very powerful in my opinion. Why not try to generalize that wonderful theorem to polynomial maps?

Well, a simple answer is: that is not possible. However: take a look at the map $F = (X + Y^2, Y)$ on \mathbb{C}^2 (i.e. $(x, y) \longrightarrow (x + y^2, y)$). Then you can see that $F^2 - 2F + I = 0$. So, F “is a zero of $T^2 - 2T + 1$ ”. The set of polynomial maps that are “zero of a polynomial $P(T)$ ” we named the “Locally Finite Polynomial Maps” (short LF).¹ Though this set was originally discovered by the speaker as a “fun” object, it slowly became clear that these maps have quite some potential: among others, they may hold the key to a better understanding of the group of invertible polynomial automorphisms!

In this talk I will introduce LF maps, give an actual extension of the Cayley-Hamilton theorem to LF maps, and pose some very puzzling questions.

¹The LF maps is a subset of the “Dynamically trivial polynomial maps”, which may make them, at least in name, less interesting to dynamical systems experts - but if they are so trivial, then please solve my questions!!