Circular choosability

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The circular chromatic number of a graph is defined as follows. A t-circular colouring of a graph G is a map $c : V(G) \to S(t)$, where $S(t)$ denotes a circle of *circumference* t, such that the length of the arc between $c(v)$ and $c(w)$ is at least one whenever $vw \in E(G)$. The circular chromatic number $\chi_c(G)$ of G is the infimum of all t for which there is a t-circular colouring. It satisfies $\chi(G) = \lceil \chi_c(G) \rceil$, where χ denotes the chromatic number, and can thus be said to be a refinement of the ordinary chromatic number. The circular chromatic number has received considerable attention since its introduction by Vince in 1988.

More recently Mohar introduced a "list version" of the circular chromatic number, the circular choosability, denoted by $\mathrm{cch}(G)$. I will give a survey of known results and open problems on circular choosability (after stating the definition properly of course). One question that I will address in particular is the following. Mohar asked for the value of

$$\tau := \sup\{\mathrm{cch}(G) : G \text{ is planar}\},$$

stating that he thought it would be between 4 and 5. I will show that, on the contrary, it is between 6 and 8.