Finding small stabilizers for unstable graphs

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In an instance of the classical, cooperative *matching game* introduced by Shapley and Shubik [Int. J. Game Theory '71] we are given an undirected graph $G = (V, E)$, and we define the value $\nu(S)$ of each subset $S \subseteq V$ as the cardinality of a maximum matching in the subgraph $G[S]$ induced by S. The *core* of such a game contains all *fair* allocations of $\nu(V)$ among the players of V, and is well-known to be non-empty iff graph G is *stable*. G is stable if its inessential vertices (those that are exposed by at least one maximum matching) form a stable set.

In this paper we study the following natural edge-deletion question: given a graph $G = (V, E)$, can we find a minimum-cardinality *stabilizer*? I.e., can we find a set F of edges whose removal from G yields a stable graph?

We show that this problem is vertex-cover hard. We then prove that there is a minimum-cardinality stabilizer that avoids some maximum-matching of G. We employ this insight to give efficient approximation algorithms for sparse graphs, and for regular graphs.