Arithmetic geometry and toric varieties: the example of Mahler's measure

Riccardo Pengo (University of Copenhagen)

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Toric varieties (say, over the complex numbers) form a vast class of examples in algebraic geometry whose behaviour can be understood by studying combinatorial objects. This makes them suitable as a test ground for many conjectures in algebraic geometry.

What happens if we start to study toric varieties over a field K that is not algebraically closed? In this talk (taking $K = \mathbb{Q}$ for simplicity) we will use the simplest examples of toric varieties (i.e. hypersurfaces in $\mathbb{G}_m^n$) and their combinatorial counterparts (i.e. the Newton polyhedron of their defining equation) to reformulate the deep conjectures of Bloch and Kato in toric terms.

This will lead us to the (still very mysterious) links between special values of L-functions and Mahler's measure, an invariant arising from diophantine approximation. We will then talk about recent work of ours aiming at to give a proof of these conjectures for elliptic curves with complex multiplication.

All the necessary number theoretical background will be introduced during the talk.