Galois geometries and Random network coding

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Presently, a new direction in coding theory, called *Random network coding*, receives a lot of attention [1].

In random network coding, information is transmitted through a network whose topology can vary. A classical example is a wireless network where users come and go.

R. Kötter and F. Kschischang proved in an inspiring article that a very good way of transmission is obtained in networks if *subspace codes* are used. Here, the codewords are k-dimensional vector subspaces of the n-dimensional vector space $V(n, q)$ over the finite field of order q.

To transmit a codeword, i.e. a k-dimensional vector space, through the network, it is sufficient to transmit a basis of this k-dimensional vector space. But a k-dimensional subspace has different bases. Kötter and Kschischang proved that the transmission can be optimalized if the nodes in the network transmit linear combinations of the incoming basis vectors of the k-dimensional subspace which represents the codeword.

These ideas led to many new interesting problems in coding theory and in Galois geometries. For instance, it leads to the study of sets C of k-dimensional subspaces of $V(n, q)$, where two different k-dimensional subspaces of C pairwise intersect in at most a t-dimensional subspace, for some specified parameter t.

Since the k-dimensional subspaces of $V(n, q)$ define $(k - 1)$-dimensional projective subspaces of the projective space $\mathrm{PG}(n - 1, q)$, this problem can also be investigated in a projective setting. Hence, Galois geometries can contribute to random network coding.

In this talk, we present a number of geometrical results on random network coding.

We concentrate on *t-intersecting constant dimension codes.* These are subspace codes consisting of k-dimensional vector spaces of a vector space $V(n, q)$, where distinct codewords pairwise intersect in t-dimensional subspaces.

The classical example of a t-intersecting constant dimension code is a *sunflower*. This is a set of k-dimensional vector spaces through a common t-dimensional subspace.

It is known that if the size of a t-intersecting constant dimension code is larger than a certain value, then the code is a sunflower. This lower bound is called the *sunflower bound*.

We first present an improvement of the sunflower bound in the case $t = 1$. We then present other results on t-intersecting constant dimension codes, including a geometrical version of a sunflower bound, and results on primitive t-intersecting constant dimension codes.

References

[1] http://www.network-coding.eu/

[2] L. Lambert, *Random network coding and designs over* $\mathbb{F}_q$. Master thesis, Ghent University, Academic year 2012-2013. Available at http://www.network-coding.eu/pubs.html