Linear codes meeting the Griesmer bound

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Abstract

A *linear* $[n, k, d]$*-code* C is a k-dimensional subspace of the vector space $V(n, q)$ of dimension n over the finite field $\mathbb{F}_q$ of order q. The vectors of C are called the *codewords* of C. The parameter d is called the *minimum distance* of the linear $[n, k, d]$-code C.

The *Griesmer bound* [4, 9] states that for a linear $[n, k, d]$-code over $\mathbb{F}_q$, the length n satisfies

$$n \geq \sum_{i=0}^{k-1} \left\lceil \frac{d}{q^i} \right\rceil = g_q(k, d).$$

Here, $\lceil x \rceil$ denotes the smallest integer greater than or equal to x.

The question arises whether there exist linear $[g_q(k, d), k, d]$-codes over $\mathbb{F}_q$, for given values of k, d and q. This is known to be possible if d is larger than some value $f(k, q)$. For instance, it is possible to construct linear codes meeting the Griesmer bound, using the construction of Belov *et al.* [1]. The construction of Belov *et al.* involves particular sets of subspaces of $\mathrm{PG}(k - 1, q)$.

To study the existence of linear $[n, k, d]$-codes meeting the Griesmer bound, we describe linear codes meeting the Griesmer bound in a geometrical way, using objects in $\mathrm{PG}(k - 1, q)$, called *minihypers* [6, 8]. These minihypers are also known in finite projective spaces under the name *blocking sets*.

We present characterization results on minihypers involving different types of objects. These include subspaces $\mathrm{PG}(t, q)$ of $\mathrm{PG}(k - 1, q)$, subgeometries defined over subfields of $\mathbb{F}_q$, and even projected subgeometries defined over subfields of $\mathbb{F}_q$, and concern both *non-weighted* as *weighted* minihypers [2, 3, 5, 7]. These characterization results lead to equivalent characterization results on linear codes meeting the Griesmer bound.

References

[1] B.I. Belov, V.N. Logachev and V.P. Sandimirov, Construction of a class of linear binary codes achieving the Varshamov-Griesmer bound, *Problems of Info. Transmission* **10** (1974), 211-217.

[2] S. Ferret and L. Storme, A classification result on weighted $\{\delta v_{\mu+1}, \delta v_\mu; N, p^3\}$-minihypers, *Discr. Appl. Math.*, to appear.

[3] P. Govaerts and L. Storme, On a particular class of minihypers and its applications. II: Improvements for q square, *J. Combin. Theory, Ser. A* **97** (2002), 369-393.

[4] J.H. Griesmer, A bound for error-correcting codes, *IBM J. Res. Develop.* **4** (1960), 532-542.

[5] N. Hamada and T. Helleseth, A characterization of some q-ary codes $(q > (h-1)^2, h \geq 3)$ meeting the Griesmer bound, *Math. Japonica* **38** (1993), 925-940.

[6] N. Hamada and T. Helleseth, Codes and minihypers. *Optimal codes and related topics.* Proceedings of the EuroWorkshop on Optimal codes and related topics (Sunny Beach, Bulgaria, June 10-16, 2001), pp. 79-84.

[7] N. Hamada and T. Maekawa, A characterization of some q-ary codes $(q > (h-1)^2, h \geq 3)$ meeting the Griesmer bound: Part 2, *Math. Japonica* **46** (1997), 241-252.

[8] N. Hamada and F. Tamari, On a geometrical method of construction of maximal t-linearly independent sets, *J. Combin. Theory, Ser. A* **25** (1978), 14-28.

[9] G. Solomon and J.J. Stiffler, Algebraically punctured cyclic codes, *Inform. and Control* **8** (1965), 170-179.