Linear codes from projective spaces

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A *linear* $[n, k]$*-code* C is a k-dimensional subspace of $V(n, q)$, where $V(n, q)$ denotes the n-dimensional vector space over the finite field $\mathbb{F}_q$.

The code C(Π) of a projective plane Π is the code generated by the incidence matrix of points and lines of Π. This code was intensively studied since the 1960's and its minimum weight and dimension is known since then (for more information, see [1]). This code was generalised by Bagchi and Inamdar in [2] to the linear code $C_{s,t}(n, q)$ of s-spaces and t-spaces in a projective space $PG(n, q)$, $q = p^h$, p prime, which is defined as the vector space spanned over $\mathbb{F}_p$ by the rows of the incidence matrix of s-spaces and t-spaces. Many questions about the codewords of small weight and dimension of $C_{s,t}(n, q)$ remain unanswered. For the dual code, even in the planar case ($s = 0$, $t = 1$), the minimum weight has not been determined in general.

After discussing the necessary background on projective spaces, we give an overview of what is known about these codes and explain some of the geometrical structures that turn up in a natural way. For example, we explain how codewords of small weight in $C_{0,t}(n, q)$ are related to a well-known geometrical concept: minimal blocking sets [6].

The study of the code C(Π) turns out to be useful for some geometrical problems; we illustrate this by the examples of the computer aided proof of the non-existence of a projective plane of order 10 by Lam, Schwiercz and Thiel [4], and the characterisation of classical unitals by Blokhuis, Brouwer, and Wilbrink [3]. We also show that the problem of the existence of hyperovals in certain projective planes and the problem of the unicity of a projective plane of prime order can be reduced to a statement about the code C(Π).

References

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