VENTING MANHOLE COVER: A NONLINEAR SPRING-MASS SYSTEM

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ABSTRACT

Manholes are vertical shafts connecting underground sewers with street-level terminals. They are covered by heavy lids. During periods of heavy rainfall, the air column in the upper part of the manhole may be compressed to such high level that the lid moves up. Of course, this is a dangerous situation for pedestrians and road traffic. Bolting the lid may provide a solution to the problem, but it is known that air pressurization underneath can result in structural damage.

A simple model is proposed to describe the lifting of the lid (manhole cover). When the lid moves up, air is allowed to escape so that the lifting pressure decreases and the lid moves down, whereupon the air pressure increases again. This repetition might lead to the realistic phenomenon of the dancing manhole cover. Since the model is strongly nonlinear, interesting dynamic behavior is expected.

Key words: manhole cover, orifice, vent, gas pocket, liquid slug, fluid-structure interaction (FSI), mass oscillation, nonlinear spring

INTRODUCTION

The dynamics of manhole covers is studied. Why are these heavy objects sometimes moving? Is it due to traffic, flooding, escaping liquid slugs (driven by compressed air as in geysering events)? What are the driving pressures and flow velocities? How to model the phenomenon?

This study is an extension of the previous paper [1], which was based on [2], and combines the models developed in [3] and [4]. The basic new elements, studied separately, are: fixed orifice and variable vent.

The gas-liquid-gas-solid interaction model is described in detail. The numerical results of two test problems are presented. Preliminary insights are discussed and tentative conclusions are drawn.
MANHOLE COVER MODEL

Manholes are covered with heavy lids (Fig. 1). The mass $m_s$ of the covers is usually sufficient to keep them in place, although sometimes they are hinged to prevent dislocation. The cover may or may not have an orifice (with diameter $D_o$) allowing pressure release (Fig. 2). The cylindrical cover is resting on a circular ring of width $(D_i - D) / 2$ and internal diameter $D_i$ from which it may be lifted off if the pressure underneath it is high enough (Fig. 3). When the cover is lifted above its depth $d_s$ gas can escape around it so that the driving pressure will decrease (Fig. 4).

The vertical displacement $x_s$ of the cover has the nonlinear constraint:

$$x_s \geq 0$$

When $x_s = 0$, the cover is at rest with gas pressure acting on the area $A = \pi D^2 / 4$ if the cover is perfectly sealed, but when the cover is displaced (i.e. $x_s > 0$) this area is increased to $A_s = \pi D_s^2 / 4$, which is another nonlinearity. It is included in the gas volume $V_1$ (see later). A more important nonlinearity occurs at $x_s = d_s$, because the vent area $A_{vent} = \pi D_s (x_s - d_s)$ becomes available for gas release. In reality the cover will tilt and rotate when it is hovering. Jet-like flow behavior may occur such that the gas velocity near the cover becomes radial with a non-uniform pressure distribution. However, herein the force on the cover is $P A_s$ for all situations displayed in Figs 2-4.

The gage pressure needed to lift the cover is $m_s g / A_s$, if the force to overcome dry friction is neglected. Otherwise the lift-off condition is:

$$P A_s \geq m_s g + \mu_s N_r$$

where $\mu_s$ is the static friction coefficient and $N_r$ is the radial normal force.
BAGNOLD MECHANISTIC MODEL

The pressure underneath the manhole cover is due to a gas pocket (labeled 1) which is compressed by an upward moving liquid column. The liquid is driven by a larger gas pocket (labeled 2) underneath it (Fig. 5). This type of problem [5-8], known as Bagnold model, consists of an oscillating liquid column to which the manhole cover responds via an entrapped gas pocket. The conduit (i.e. manhole) is closed at the bottom (at \( x = 0 \)) and has a cover as described above (Figs 2-4) at the top (at \( x = x_L \)). The liquid column of length \( L_0 \) is in between \( x = x_2(t) \) and \( x = x_1(t) \). Spring stiffness and damping may be added, but are zero in the test problems (\( k_s = 0 \) and \( c_s = 0 \)). The manhole may have a (small) inclination, but is vertical herein (\( \theta = -\pi/2 \)). The vent can be in the conduit wall (Fig. 5), around the cover (Fig. 4) or within the cover (Figs 2-3).

Fig. 5 Model of manhole with cover and vent (with \( \theta = -\pi/2 \)).
[3] Figure to be rotated anti-clockwise by 90°.

GOVERNING EQUATIONS

The two gas columns (labeled 1 and 2) contain perfect gas of uniform pressure and the rigid liquid column behaves like a plunger. The seven variables describing the gas-liquid-gas-solid system shown in Fig. 5 are: liquid velocity \( v \), liquid front position \( x_1 \), gas absolute pressure \( P_1 \), driving absolute pressure \( P_2 \), solid velocity \( v_s \), solid displacement \( x_s \) (relative to the length \( x_L \)) and gas mass \( m_1 \). They have to satisfy seven coupled ordinary differential equations [1, 3]:

\[
\frac{dv}{dt} = \frac{P_2 - P_1}{\rho L_0} + g \sin \theta - \frac{f}{2D} v |v| \tag{3}
\]

\[
\frac{dx_1}{dt} = v \tag{4}
\]

\[
\frac{dP_1}{dt} = \frac{n_1}{x_L + \alpha x_s - x_1} \left( v - \alpha v_s \right) + \frac{n_1}{m_1} \frac{dm_1}{dt} \tag{5}
\]

\[
\frac{dP_2}{dt} = -\frac{n_2}{x_1 - L_0} P_2 \frac{v}{x_1 - L_0} \tag{6}
\]

\[
\frac{dv}{dt} = A_s \left( \frac{P_2 - P_{\text{atm}}}{m_s} \right) + g \sin \theta - k_s \frac{v}{m_s} x_s - c_s v \frac{F_s}{m_s} \cdot \text{sign}(v) \tag{7}
\]

\[
\frac{dx}{dt} = v_s \tag{8}
\]

\[
\frac{dm_1}{dt} = \begin{cases} \frac{-C_d A^+ Y(P_1) \sqrt{2 \rho_1 (P_1 - P_{\text{atm}})}}{P_{1,0}} & \text{if } \frac{P_1}{P_{1,0}} \leq 1.89 \\ \frac{-C_d A^+ \frac{k+1}{k-1} \sqrt{P_1 P_2}}{P_{1,0}} & \text{if } \frac{P_1}{P_{1,0}} > 1.89 \end{cases} \tag{9}
\]

with

\[
A^+ = \begin{cases} A_{\text{or}} = \pi D_{\text{or}}^2 & \text{if } D_{\text{or}} > 0 \\ 0 & \text{if } D_{\text{or}} = 0 \text{ and } x_s \leq d_s \\ \pi D_s (x_s - d_s) & \text{if } D_{\text{or}} = 0 \text{ and } x_s > d_s \end{cases} \tag{10}
\]

and

\[
Y(P_1) = \begin{cases} 1 - \frac{\frac{P_{1,0}}{P_1^k}}{\frac{P_{1,0}}{P_1^{k-1}}} & \frac{1}{2} \text{ if } \frac{P_1}{P_{1,0}} \leq 1.89 \\ \frac{k-1}{k} \left( \frac{P_1}{P_{1,0}} \right)^2 \left[ 1 - \frac{\frac{P_{1,0}}{P_1^k}}{\frac{P_{1,0}}{P_1^{k-1}}} \right] \text{ otherwise} \end{cases} \tag{11}
\]

Here \( \alpha = A_s / A \) accounts for the diameter change in the calculation of the gas volume \( V_1 \) (Eq. 12) if \( x_s > 0 \), \( C_d \) is the orifice or vent discharge coefficient, \( f \) is the skin friction coefficient for the moving liquid slug, \( F_s \) is the Coulomb friction force acting on the vertical sides of the cover, \( \rho \) is the mass density of the liquid, \( \rho_1 \) is the mass density of the upper gas, \( g \) is the gravitational acceleration, \( n \) is the gas polytropic coefficient, \( R \) is the specific gas constant, \( P_{\text{atm}} \) is the atmospheric pressure outside the manhole, and \( \theta \) is the downward angle. If \( x_s = 0 \) then \( v_s = 0 \) and the left-hand side of Eq. (7) is replaced by \( -N_r / m \), with \( N_r \) the supporting ring’s force acting on the cover. The gas escape area \( A^+ \) is either constant (\( A_{\text{or}} \)) or variable (\( A_{\text{vent}} \)).

The driving pressure \( P_2 \) may be either constant (as \( P_{2,0} \) by taking \( n_2 = 0 \)) or variable (herein). The starting position of the slug front is \( x_{1,0} = x_{2,0} + L_0 \), the initial pressures are \( P_{1,0} \) and \( P_{2,0} \).
the initial velocities \( v_0 \) and \( v_{s,0} \), and the initial gas mass \( m_1 = m_{1,0} \).

The cover’s initial position is \( x_{s,0} = 0 \) at \( x = x_L \).

The responding pressure \( P_1 \) in the venting gas pocket (labeled 1) is obtained (Eq. 5) from a combination of its adiabatic compression / expansion and its loss of mass (Eq. 9) according to [2, 4]. Back flow through vent or orifice is (currently) not modeled. The auxiliary variables are volume, density and temperature of the gas in the venting pocket:

\[
V_1 = A (x_L - x_1) + A_s x_s \quad (12)
\]

\[
\rho_1 = \frac{m_1}{V_1} \quad (13)
\]

\[
T_1 = \frac{P_1}{\bar{R} \rho_1} \quad (14)
\]

**NUMERICAL SOLUTION**

The governing equations (3-9) constitute one autonomous nonlinear first-order ordinary differential equation:

\[
\frac{dy}{dt} = f(y), \quad \text{with} \quad y := \begin{bmatrix} v \\ x_1 \\ P_1 \\ P_2 \\ v_s \\ x_s \\ m_1 \end{bmatrix} \quad (15)
\]

Note that the vector function \( f \) itself depends on the variables in \( y \). That is, four different functions \( f \) are used depending on \( P_1 \) (discriminating non-choking and choking flow) and \( x_s \) (discriminating stationary, lifted and venting position of cover).

Heun’s method has been used to solve Eq. (15) with a numerical time-step \( \Delta t = 1 \) ms.

**TEST PROBLEM WITH CONSTANT ORIFICE**

The data for manhole and cover, water and air, have been taken from [9] and [1], but now the cover has an orifice in it (Fig. 3). The system parameters are: shaft \( D = 0.5 \) m, cover \( D_s = 0.55 \) m, so that \( \alpha = 1.21 \), depth \( d_r = 0.055 \) m, liquid \( L_0 = 25 \) m, upper gas \( L_{1,0} = 75 \) m, lower gas \( L_{2,0} = 50 \) m (so that volume \( V_{2,0} = 9.8 \) m³), total length \( x_L = L_{2,0} + L_0 + L_{1,0} = 150 \) m, \( P_{2,0} = 3.7 \) bar absolute, \( P_{1,0} = P_{\text{atm}} = 1 \) bar absolute, \( n_1 = n_2 = 1.2 \), \( \rho = 1000 \) kg/m³, \( f = 0.02 \), and \( \theta = -\pi/2 \). The manhole cover has mass \( m_s = 100 \) kg and its initial position is \( x_{s,0} = 0 \). Furthermore \( k_s = 0, c_s = 0 \) and \( F_s = 0 \). There is no initial motion: \( v_{s,0} = 0 \) and \( v_0 = 0 \). Further, \( C_1 = 1, m_{1,0} = 17.7 \) kg, \( \rho_{1,0} = 1.2 \) kg/m³, \( T_{1,0} = 290 \) K, \( R = 287 \) m²/s/K, \( k = 1.4 \), and \( g = 9.81 \) m/s². The orifice size \( D_{or} = 0.02D = 10 \) mm and the cover moves up and down without losing air at its circumference.

The initial situation is not in equilibrium and the liquid column will move up if \( P_2 - P_1 > \rho L_0 g \) \( = 2.45 \) bar, thereby compressing gas column 1 and decompressing gas column 2. An absolute pressure \( P_1 = P_s = 1.041 \) bar is needed to lift the cover mass from its supporting ring (based on lifting area \( A_s \)).

The cover will move vertically up and down according to Eq. (7). The calculated water velocity \( v \) and air pressure \( P_1 \) are shown in Figs 6 and 7. The higher frequency of oscillation of the air pressure \( P_1 \), compared to the frequency of the driving water column’s velocity \( v \), is caused by the upward motion of the manhole cover (FSI). The monotonically decreasing air mass \( m_1 \) is shown in Fig. 8. The cover velocity and displacement, \( v \) and \( x_s \), are shown in Figs 9 and 10, where Fig. 11 is the corresponding phase diagram. The driving pressure \( P_2 = 3.7 \) bar is large enough to lift up the cover more than 2 meters. The orifice adds – next to skin friction acting on the liquid column – damping to the system by releasing the gas.

**Fig 6 Water velocity (\( D_{or} / D = 0.02 \)).**

**Fig 7 Air pressure (\( D_{or} / D = 0.02 \)).**
**TEST PROBLEM WITH VARIABLE VENT**

The input data are the same as before, with $D_{or} = 0$, but now the air will vent when the cover is lifted up far enough (when $x_s > d_e$), see Fig. 4. From the calculated air pressure, air mass and cover displacement in the Figs 12, 13 and 14, it is seen that the manhole gives one burp, after which the upper pressure remains too low ($P_1 \leq P_s$) to lift the cover again. The pressure $P_1$ even comes sub-atmospheric periodically. There is no “dancing” at all. However, the maximum cover displacement is more realistic with the vent added: $x_{s,\text{max}} - d_e = 62 \text{ mm above street level}$.

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**Fig. 8** Air mass ($D_{or} / D = 0.02$).

**Fig. 9** Cover velocity ($D_{or} / D = 0.02$).

**Fig. 10** Cover displacement ($D_{or} / D = 0.02$).

**Fig. 11** Cover velocity versus cover displacement ($D_{or} / D = 0.02$).

**Fig. 12** Air pressure ($D_{or} = 0$, circumferential vent, $P_2 = 3.7$ bar; $L_0 = 25 \text{ m}$, upper gas $L_{1,0} = 75 \text{ m}$).
When the manhole is initially nearly “empty” (filled with air), say liquid $L_0 = 10$ m, upper gas $L_{1,0} = 90$ m, the results displayed in Figs 15, 16 and 17 are obtained. The cover will do a little dancing: it bounces up and down six times, with a maximum elevation of 0.28 m above street level.

DISCUSSION

The proposed model is delicate because of its nonlinearities and geometrical discontinuities. The calculations stop when either nearly all air has escaped (i.e. 5% volume left [4]) or back flow occurs in orifice or vent ($P_1 < P_{atm}$). The manhole cover simply blows off if the driving pressure $P_2$ is too large or it stays at rest if $P_2$ is too small. If venting occurs ($x_s > d_s$) then the compressed air is quickly released. It is not certain whether the current Bagnold model can produce “dancing” manhole covers, which might be a resonance phenomenon. One could do an inverse analysis: take recorded or hypothetical cover displacements and determine which driving pressures belong to it.

CONCLUSION

A basic and preliminary study of the interaction of an accelerating water column with an entrapped air column and a displacing manhole cover has been presented. The emphasis was on fluid-structure interaction and air venting. A relatively simple mechanistic model already gives plenty food for thought.
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NOMENCLATURE

\[ A = \text{cross-sectional manhole area (m}^2) \]
\[ A^+ = \text{gas escape area (m}^2) \]
\[ A_{or} = \text{orifice area (m}^2) \]
\[ A_s = \text{manhole cover area (m}^2) \]
\[ A_{vent} = \text{venting area (m}^2) \]
\[ C_d = \text{orifice discharge coefficient} \]
\[ c_s = \text{dashpot coefficient (kg/s)} \]
\[ D = \text{manhole diameter (m)} \]
\[ D_{or} = \text{orifice diameter (m)} \]
\[ D_s = \text{manhole cover diameter (m)} \]
\[ d = \text{depth of manhole cover (m)} \]
\[ F_s = \text{Coulomb friction force (N)} \]
\[ f = \text{Darcy-Weisbach friction coefficient} \]
\[ f = \text{vector function} \]
\[ g = \text{acceleration due to gravity (m/s}^2) \]
\[ k = \text{adiabatic constant, ratio of specific heats} \]
\[ k_s = \text{spring stiffness (N/m)} \]
\[ L = \text{length of gas column (m)} \]
\[ L_0 = \text{length of liquid column (m)} \]
\[ m_s = \text{mass of manhole cover (kg)} \]
\[ m_1 = \text{mass of gas (kg)} \]
\[ N = \text{normal force (N)} \]
\[ n = \text{constant polytropic exponent} \]
\[ P = \text{gage pressure (Pa)} \]
\[ P_r = \text{absolute pressure needed to lift the cover (Pa)} \]
\[ P_1 = \text{absolute pressure of responding gas (Pa)} \]
\[ P_2 = \text{absolute pressure of driving gas (Pa)} \]
\[ R = \text{specific gas constant (m}^2/(s}^2\text{K}) \]
\[ T = \text{absolute temperature (K)} \]
\[ t = \text{time (s)} \]
\[ V = \text{volume of gas (m}^3) \]
\[ v = \text{velocity of liquid column (m/s)} \]
\[ v_s = \text{velocity of manhole cover (m/s)} \]
\[ x = \text{distance along central axis of manhole (m)} \]
\[ x_s = \text{length of manhole (m)} \]
\[ x_s = \text{displacement of manhole cover (m)} \]
\[ x_1 = \text{position of liquid front (m)} \]
\[ x_2 = \text{position of liquid tail (m)} \]
\[ Y = \text{adiabatic gas expansion factor} \]
\[ y = \text{vector of unknowns} \]
\[ \alpha = \text{ratio } A_s/A \]
\[ \theta = \text{angle of downward inclination of manhole (rad)} \]
\[ \mu_s = \text{static friction coefficient} \]
\[ \rho = \text{mass density of liquid (kg/m}^3) \]
\[ \rho_1 = \text{mass density of gas (kg/m}^3) \]

Subscripts

atm = atmospheric
max = maximum
or = orifice
r = radial direction
s = solid, static
x = axial direction
0 = constant, initial value
1 = responding gas
2 = driving gas

REFERENCES