Johannes von Kries and the History of Water Hammer

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Introduction

Water hammer in pipe systems encompasses a range of phenomena familiar to engineers, including (1) the propagation of disturbances created by valve operations, turbomachinery regulation, and accidents, through waves traveling at the speed of sound because of the compressibility of the liquid and the elasticity of the pipe walls; (2) the reflection of these waves off pipe-end boundaries and internal features, such as changes in cross section or branches, leading to characteristic frequencies or periods of oscillation; (3) large pressure changes arising from fluid velocity changes, especially in liquids, because of their high density and low compressibility; and (4) associated unsteady flow phenomena, such as column separation and gas release at low pressures or inertial mass oscillations associated with free surfaces. The propagation and reflection of pressure waves are of interest in many other fields, such as in acoustics, hemodynamics, and solid mechanics, but the need to calculate the high pressure rises and pipe-wall stresses characteristic of relatively rigid pipes carrying liquids is the focus of water hammer in hydraulic engineering. Only in hemodynamics is there a similar interest in pressures and tube wall stresses, even though they tend to be lower in flexible systems. Historically, therefore, the development of what is now widely called the *Joukowsky equation* defines the origins of a water-hammer theory that is appropriate for engineering practice.

This paper reviews the history of the Joukowsky equation and the forgotten contributions by Johannes von Kries. The water-hammer community is ignorant of the work of Kries: for example, the extensive reviews by Wood (1970), Thorley (1976), and Ghidaoui et al. (2005) do not mention them; and the reviews by Boulanger (1913), Lambossy (1950), and Stecki and Davis (1986) briefly refer to Kries's 1892 book only. In the hemodynamics community, Kries found some recognition (Frank 1927; Evans 1962; Bernstein and Evans 1962; Kenner 1972; Sperling et al. 1975).

Researchers of water hammer highlighted in historical papers include the following: Joukowsky, portrayed by Golubev (1947), Strizhevsky (1957), and Stepanov (1997); Gariel (and water-hammer research before 1914), reviewed by Réméniéras (1961); Ménabréa, translated by Anderson (1976); Michaud, commemorated by Almeida (1979); and Allievi, commemorated by Franke (1992) and Ceccarelli (1999). Skalak (1999) wrote an "*in memoriam*" on his father, Hager (2001) celebrated Schnyder and Jaeger, and Wiggert and Wylie (2003) paid a tribute to Streeter. This paper on Johannes von Kries is a further contribution to archiving the history of water hammer. It briefly describes the work and life of Kries and it reviews his research on blood flow, his 1883 paper, and his 1892 book. His contribution to the knowledge of water hammer is placed in the historical context of transients in fluids and solids.

Joukowsky Equation for Fluids

The fundamental equation in water-hammer theory relates pressure changes, Δp, to velocity changes, Δv, according to

$$\Delta p = \rho c \Delta v \tag{1}$$

where ρ=fluid mass density; and c=speed of sound. Korteweg's (1878) formula defines c for fluid contained in cylindrical pipes of circular cross section

$$c = \sqrt{K^*/\rho} \quad \text{and} \quad K^* = K/[1 + (DK)/(eE)] \tag{2}$$

where D=diameter of the pipe; e=wall thickness; E=modulus of elasticity for the wall; and K=bulk modulus of the contained fluid.

Relation (1) is commonly known as the *Joukowsky equation*, but it is sometimes called either the *Joukowsky–Frizell* or the *Allievi equation*. Its first explicit statement in the context of water hammer is usually attributed to Joukowsky (1898). Frizell (1898) and Allievi (1902, 1913), unaware of the achievements by Joukowsky and Frizell, also found Eq. (1), but they did not provide any experimental validation. Anderson (2000) noted that Rankine (1870) had already derived Eq. (1) in a context more general than water hammer. Kries (1883) derived relation (1), mentioning—without a particular reference—its existence in the theory of shock waves but at the same time stating that it had not been validated by experiments, something that he would do.

There is a parallel between the contemporaries, Joukowsky (1847–1921) and Kries (1853–1928). Both are famous because of their work in other fields: Joukowsky in aerodynamics and Kries in physiology. Both of their investigations on water hammer are impressive because of their clarity and maturity in theory and in experiment. The transient event of water hammer was difficult to capture in their day. Joukowsky measured fast waves in long steel pipes, and Kries measured slow waves in short rubber hoses; so their test systems had relatively large times L/c, where L=length of the tube.

Why is Eq. (1) not the Kries equation, or at least the Joukowsky–Frizell–Kries (JFK) equation? That Kries's interest was in blood flow is not an excuse, because much of the other and better known nineteenth-century literature was also on blood flow (see, e.g., Jouguet 1914), starting with Young's obscure paper dated 1808. It has probably more to do with dissemination. Joukowsky's paper presented in 1898—published in Russian in 1899 and published in German in 1900—has been (partly) translated into English by Simin (1904) and (partly) into French by Goupil (1907). Furthermore, Rouse and Ince (1957) declare

Joukowsky to be the founder of the water-hammer discipline. The work of Kries on blood flow was all in German and has never been translated.

The Joukowsky Equation for Solids

The early investigators of water hammer had not noticed the analogy with longitudinal waves in solid bars (Boulanger 1913), except for Stromeyer (1901) in a rare paper and Gibson (1908). Young (1807) found that the strain ε produced by the impact of elastic solid bodies equals v/c. With Hooke's law stating that $\varepsilon=-\sigma/E$, where σ=stress and E=Young's modulus of elasticity, this gives $\sigma=-Ev/c$. Assuming that $c=\sqrt{E/\rho}$, one obtains for the solids equivalent of Eq. (1)

$$\sigma = -\rho c v \tag{3}$$

Young (1808) was the first to find the pressure wave speed for incompressible liquids contained in elastic tubes, and the writers believe that Young was also aware of the speed of sound in solid bars, $c=\sqrt{E/\rho}$. Young's work is difficult to read, but Timoshenko (1953, pp. 93–94) gives a neat summary. It is noted that the strain ε in liquids contained in tubes equals p/K^*, where K^*=effective bulk modulus representing the combined effects of fluid compressibility and tube wall elasticity.

Saint-Venant (1867) gives a clear, rigorous, and complete treatment of the longitudinal collision of two solid bars. This collision is analogous to frictionless water hammer. He derives, for a bar of cross section A, $F=\sigma A=-EA\varepsilon$, $v=c\varepsilon$, and $c=\sqrt{E/\rho}$, which can be combined into Eq. (3). In later papers, Saint-Venant (1870, 1883) gives full credit to Babinet for the first clear derivation of c (oral presentation in 1829, written down by Pierre in 1862), although the formula itself goes back to Newton, Euler, and Lagrange. The corresponding speed of sound in liquids is $c=\sqrt{K^*/\rho}$. Korteweg (1878) derived the proper value for K^* in water hammer, given in Eq. (2). Saint-Venant also employed a graphical method foreshadowing the Schnyder (1932)–Bergeron (1935) graphical method (this method was the standard water-hammer calculation tool in the precomputer era). It is remarkable that Rankine (1867) reviewed Saint-Venant's (1867) paper (with partial translation into English). In previous work, Rankine (1851) had found the wave speed of nearly longitudinal vibration and he had already noted the similarity of vibrations in solids and liquids. Joukowsky (1878) also studied the impact of solid bodies; however, without considering the elasticity of the material.

The history of this subject is extensively described by Todhunter and Pearson (1886, 1893) and Timoshenko (1953). Timoshenko and Goodier (1970) summarize the achievements of Young and Saint-Venant. Bergeron (1950) is probably the first to apply—the other way around—water-hammer theory to the axial vibration of solid bars.

Work and Life of Kries

Johannes von Kries (Fig. 1) was one of the big names in the late nineteenth and early twentieth centuries. He contributed to the areas of physiology, psychology, philosophy, mathematics, and law. He is best known for his physiological work, in particular for his studies of the sense of vision. The sense of hearing, nerves and muscle mechanics, hemodynamics, the theory of probability—and more—are subjects of his 121 publications. His life and his scientific achievements have been meticulously described in dissertations by Oser (1983) and Lorenz (1996). Hoffmann (1957) focused on the philosophical side of Kries, and the view of Kries on his own life and work can be found in Grote (1925).

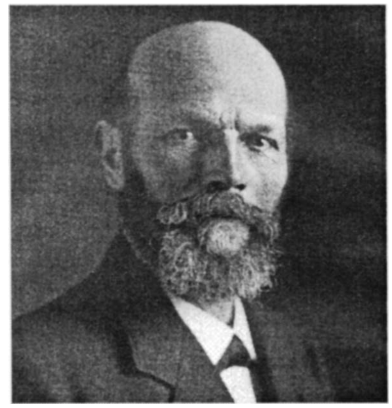

Fig. 1. Johannes von Kries (1853–1928) (Grote 1925)

Johannes von Kries was born on October 6, 1853, in Roggenhausen, Prussia. In 1869, at the age of 16, he started studies in medicine at the University of Halle under the physiologist Richard von Volkmann. He continued his studies at Leipzig under the physiologist Karl Ludwig and the mathematician Karl Neumann, as well as at Zurich, and finally finished as a doctor of medicine at Leipzig in 1875. After one year of military service, he became from 1876 to 1877 a voluntary postdoctoral researcher at the Institute of Physics of the University of Berlin under Hermann von Helmholtz. [Although it is possibly only a coincidence, Helmholtz had interests in both acoustics and hemodynamics and was credited by both Korteweg (1878) and Joukowsky (1898) with first suggesting that the sonic wave speed in pipes is influenced by both fluid and pipe-wall elasticities. Helmholtz (1848) and André (1870) came up with that suggestion in an experimental study of wave speeds.] In 1877, Kries became assistant to Karl Ludwig at Leipzig. He attained "Habilitation" (the status of university lecturer) in physiology in 1878 and was a private teacher in Leipzig from 1878 to 1880. In 1880, he became extraordinary professor of physiology at Freiburg and in 1883 became full professor and director of the physiological institute, from which he retired in 1924. Kries was cofounder of the *Zeitschrift für Psychologie* with Ebbinghaus and was one of its first editors. He received the German order *Pour le mérite* in 1918 and three honorary doctorates. Johannes von Kries died on December 30, 1928, in Freiburg, Germany, at the age of 75.

Kries's Work on Blood Flow

Kries's first publication on hemodynamics appeared in 1878. It describes the manometer measurement of the average blood pressure. In 1883, his memorable paper, summarized in the following section, was presented. Two papers in 1887 (a, b) presented an improvement of existing techniques to measure the pulse in human bodies. The measuring device is sketched in Fig. 2(a). A person's forearm (or foot) is enclosed in a narrow container filled with air (or with gasoline vapor for better results). The in-and-out flow of blood to the forearm makes its volume change. Air, thus driven in and out of the container, feeds the flame. The time-varying flow in the arteries typically lets a flame increase from 3

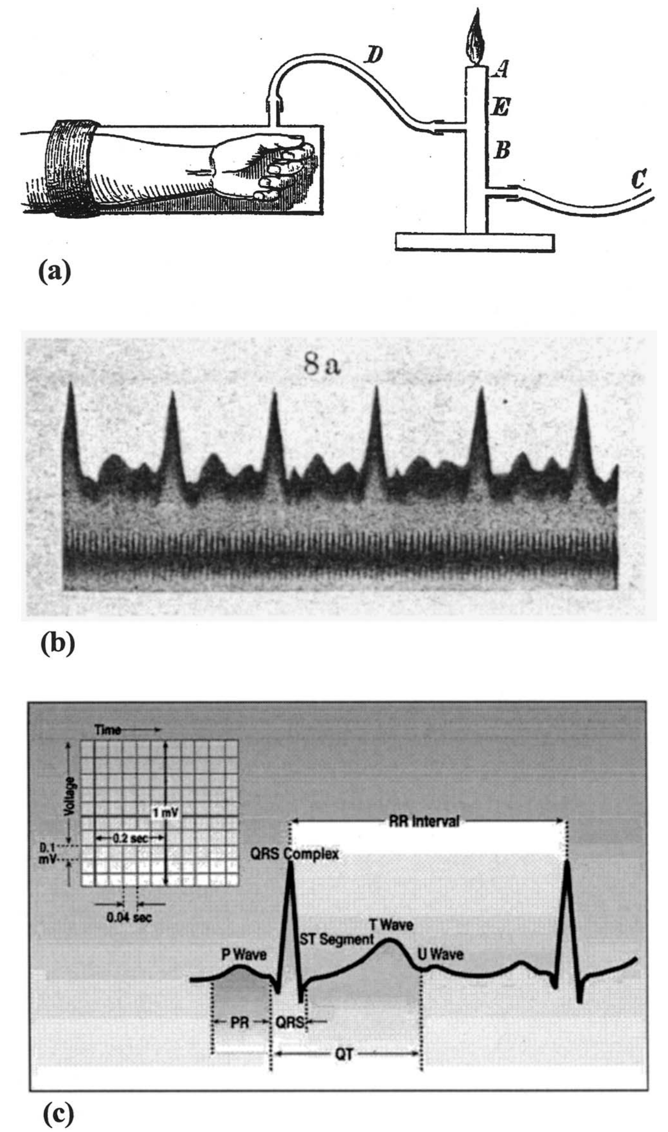

Fig. 2. (a) "Flammentachometer" (flame velocity meter) (Kries 1887a); (b) ancient tachogram (Kries 1892); and (c) modern electrocardiogram [Frank G. Yanowitz, Univ. of Utah School of Medicine, ⟨http://medstat.med.utah.edu/kw/ecg/⟩ (1997), used with permission]

cm to 4 to 10 cm height. The flow variations were directly related to pressure variations through Eq. (1). Kries obtained photographic records of the pulse, which he called *tachograms*. Fig. 2(b) shows such a tachogram, and for comparison, Fig. 2(c) displays a modern electrocardiogram. A third paper in 1887(c) recognized that the maximum velocity (at the central axis) in laminar pipe flow is twice the cross-sectionally averaged velocity. To verify this theoretical result, Kries carried out accurate tests with water and with milk. The experiment involved measuring length and volume, but not time. The issue was of importance in estimating blood circulation times in arterial systems. All his previous work on blood flow was incorporated in his book (1892), to be described subsequently. The last published contribution on the human pulse is Kries (1911).

Kries's 1883 Paper

In the introduction of Kries (1883), the state of knowledge of the pulse is described. Much experimental data existed, but a proper theoretical background was missing. He was aware of previous contributions by Young (1808), Weber (1866), Résal (1876), Korteweg (1878), and others; but he believed that all these studies were not of much interest to physiologists because they focused just on one aspect: a theoretical value for the wave propagation speed. Kries wanted to go beyond that aspect. He mentioned the equivalence of incompressible fluid in an elastic tube (pulse) and compressible fluid in a rigid tube (water hammer). His one-dimensional model for linear wave propagation described both.

The first part of Section I presented the basic theory of water hammer, including the Joukowsky formula. Kries correctly assumed uniform pressure in the radial direction, cross-sectional averages of axial velocities, hoop stress in the tube wall proportional to pressure, negligible influence of convective terms, and Moens–Korteweg wave propagation speed, noting that there might be a dependency on pressure in flexible rubber hoses. The continuity and the dynamic equations were combined into a classical wave equation, which had D'Alembert traveling-wave solutions for pressure and velocity. The derivation of the pressure–velocity relation Eq. (1) followed then from basic principles. Kries stated that an analogue of this relation was already known, but not validated, in the theory of sound waves in air.

The second part of Section I described the experimental validation. A constant-head reservoir supplied water to a 4 to 5 m long, thin-walled, rubber hose of 5 mm diameter. The steady discharge was measured. Rapid valve closure caused a pressure rise, measured with a spring-manometer, as shown in Fig. 3. The valve closed at time s in Fig. 3, and the reflection from the reservoir arrived at time r. The wave speed, c, was estimated from the reflection time, $2L/c$. Measured pressure rises (in mmHg) for three different flow rates were 31.1, 50.0, and 72.0; the corresponding values according to Eq. (1) were 29.9, 47.6, and 71.6. Experiments in a tube with a more rigid wall gave values of $\Delta p = 69.0$ mmHg and $\rho c \Delta v = 70.0$ mmHg. Content with this validation, Kries considered periodic velocity-excitation of an infinitely long tube (no reflections), giving the classical square wave in Fig. 4.

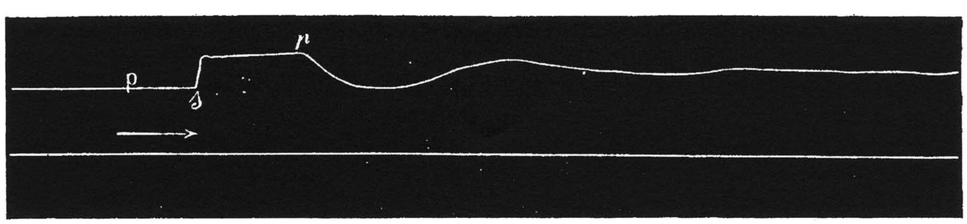

Fig. 3. Measured Joukowsky pressure in large-diameter hose; p=pressure, s=closure, r=reflection; and $r - s \approx 0.6$ s (Kries 1883)

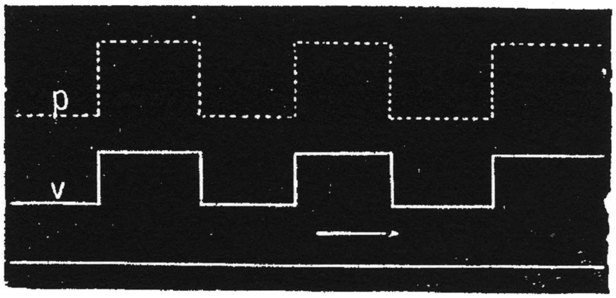

Fig. 4. Theoretical pressure and described velocity changes in an infinitely long tube for a frictionless fluid: p=pressure; and v=velocity (Kries 1883)

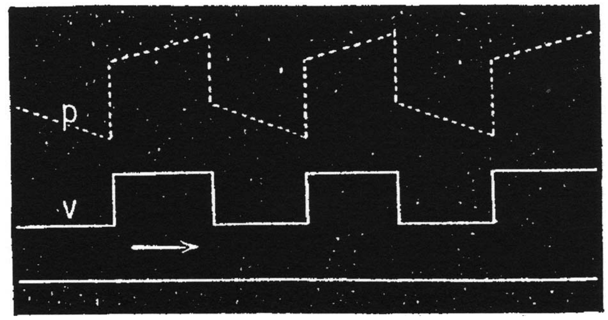

Fig. 6. Theoretical pressure and described velocity changes in an infinitely long tube, with friction: p=pressure; and v=velocity (Kries 1883)

The first part of Section II developed the theory for water hammer with linear friction. The friction term, added to the dynamic equation, is taken proportional to the flow velocity. Kries mentioned that the constant of proportionality, η, depends on fluid properties and tube diameter, but he did not specify its value as given for laminar pipe flow by Hagen (1839) and Poiseuille (1840) [see (Brown 2002)]. In the same year, 1883, Gromeka modeled the same friction term, thus making his more-advanced equations too difficult to solve.

Kries ended up with the telegrapher's equation, which he subjected to a harmonic analysis. For small friction terms, he derived constant values for wave damping and phase velocity [the Moens–Korteweg wave speed, Eq. (2)]. The friction term caused small differences in phase and amplitude in the relation between velocity and pressure; note that pressure and velocity are in phase in the frictionless case shown in Fig. 4. These small differences were not constant but were frequency dependent. The changed amplitude, caused by friction, of the pressure in the harmonic solution was used to predict line pack. After some manipulation, line pack at the valve was estimated from

$$\frac{dp_{\text{linepack}}}{dt} \approx \frac{1}{4}\eta\rho c v \qquad (4)$$

where $\eta = 32\nu / D^2$, according to the Hagen–Poiseuille law, where ν=fluid kinematic viscosity and D=inner tube diameter. However, the present authors are not entirely convinced by Kries's derivation of formula (4). A better explanation, in terms of the initial (steady-state) pressure gradient, as well as more examples, was given in his book (Kries 1892). Joukowsky (1898) also recognized that line pack is the consequence of an initial pressure gradient, and he explained it by means of a spatially stepwise increasing initial pressure (Joukowsky 1898). Joukowsky (1898) measured line pack in steel pipes and proposed a formula similar to Eq. (4), as did Gibson (1930).

The second part of Section II concerns the first observation of line pack and the verification of Eq. (4). Fig. 5 shows the result of an experiment in a narrow tube. The line-pack effect makes the pressure slowly rise after the rapid valve closure at time s. For tubes of increasing resistance, Kries measured line pack values of 18.3, 40.0, and 72.9 mmHg/s. The corresponding values calculated from Eq. (4) were 19.6, 38.3, and 73.1 mmHg/s. Theoretical results for an infinitely long tube, excited by periodic velocity-pulses at one point, are sketched in Fig. 6.

Section III discusses pressure pulse and blood flow in the aorta. Backed up by his experiments, Kries stated that friction is unimportant in the aorta. The beating heart induces flow velocity changes that directly relate to the pressure pulse through Eq. (1). He also considered the longitudinal stretching of the aorta wall and explained the possibility of a secondary pressure rise, shown in Fig. 7, because of axial motion of the closed heart-valve. This is one of the first examples of fluid–structure interaction, today called *junction coupling*. Kries concluded that many other secondary effects in the vascular system existed for which a theoretical background was absent, for example, wave reflections from ends that are neither open nor closed.

Section IV deals with the (im)possibilities of measuring pressure pulse and volume flow in peripheral arteries. Theoretically, reflected waves can be distinguished in a signal if pressure and velocity are measured at the same location. Such a simultaneous measurement could not be done with sufficient reliability in 1883.

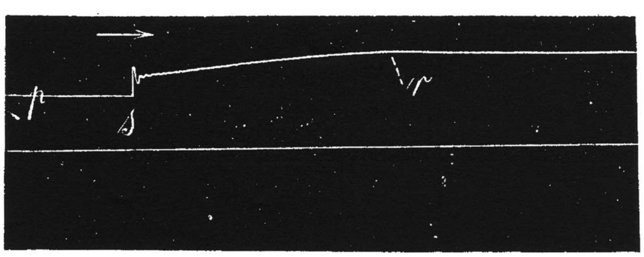

Fig. 5. Measured Joukowsky pressure and line pack in small-diameter hose; p=pressure, s=closure, r=reflection; and $r-s\approx 1.5$ s (Kries 1883)

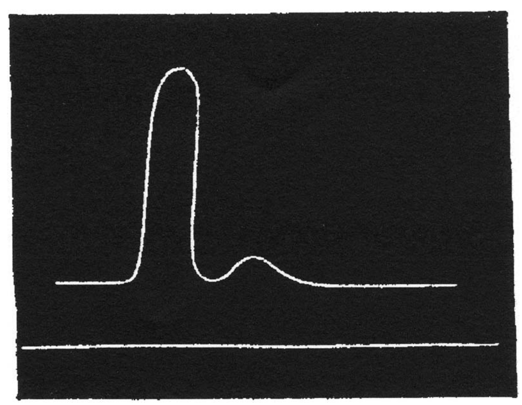

Fig. 7. Sketch of pressure pulse in stretching aorta, with fluid-structure interaction (Kries 1883)

Kries's 1892 Book

Kries's book (1892) is well written and a pleasure to read. It is based on his 1883 paper; but the material is improved, extended, and presented in a more structured way. He uses the theory developed in Chapter I to explain the pulse in the remaining chapters. Chapter II considers the form of the pulse and the dicrotic wave (secondary pulse), Chapter III deals with the aortic bifurcation, and Chapter IV discusses various aspects (gravity, temperature) that affect the pulse. The appendixes give the mathematical theory. The book displays many experimental results: manometer measurements in rubber hoses and accurate pulse records with the flammentachometer shown in Figs. 2(a and b).

The fundamental Chapter I gives the general theory of traveling waves, standing waves, attenuation, reflection, forced oscillation, tube breathing, and ovalization. The theory was applied to rubber hoses; and with respect to these, Kries observed viscoelastic retardation of the wall material from careful tests. He understood the phenomenon; it explained the fact that Weber (1866) measured wave speeds that were 12% larger than the theoretical predictions. The internal pressure may influence the wave speed, because it changes the tube diameter and the wall properties (like stiffness). The latter is the case for tubes made from intestine membranes. He validated Eq. (1), now through tests with fluid injection instead of valve closure. His treatment of wave attenuation caused by skin friction is an elaboration of his 1883 work. He found exponential damping of the waveform; and for systems with much friction, waveform distortion because of frequency-dependent phase velocities. Slow pressure variations have low phase-velocity and low damping; fast pressure variations have high phase-velocity (about the Moens–Korteweg wave speed) and high damping. Line pack is correctly explained, but Eq. (4) is absent from the book. Appendix IV, giving formulas for frequency-dependent damping and phase-velocity, can be seen as the first step toward investigating unsteady friction. The frequency-dependent phase-velocity, $c(\omega)$, follows from

$$\frac{1}{[c(\omega)]^2} = \frac{1}{2c^2}\left(1 + \sqrt{1 + \frac{\eta^2}{\omega^2}}\right) \qquad (5)$$

where ω=circular frequency; $\eta = 32v/D^2$; and c=Moens–Korteweg wave speed, Eq. (2). Kries's theoretical studies of reflections from open ends, closed ends, branches, tapered sections, and abrupt changes of friction were supported by experimental results. Kries wondered how the pulse, traveling from the aorta into the many branches of the arterial system, could occur without reflections. As a result, he derived the condition for reflection-free multitube branches. The fundamental water-hammer periods $2L/c$ and $4L/c$ appear in his section on traveling and standing waves in finite-length tubes. In an explanation of experimental results by Moens (1878), he produced the midpoint pressure history shown in Fig. 8, which is typical for water hammer in a single tube. Kries discussed the possible evidence of wall vibration in measured pressure histories. First, he considered "hoop" vibration in which the wall remains circular. He found that the corresponding "ring" period, $\pi D/c_{\text{wall}}$, was much too small to be measured. Hoop vibration changes the *size* of the cross-sectional area and thus causes axial flow and axial wave propagation in fluid and wall. Second, he considered wall vibration changing the *form* of the cross-sectional area, but preserving its size. Axial interaction is less in this case and the fundamental periods are larger. The deformation (Fig. 9), caused by internal hydrostatic pressure, of flexible tubes laid on a flat floor was studied theoretically; and the associated oscillation was examined in tests with bouncing tubes. The period of oscillation of an ovalizing tube (Fig. 10) was estimated from a formula originally derived by Rayleigh for the capillary oscillation of free jets. The theoretical predictions were confirmed by experimental results obtained in free-hanging tubes in Appendix VIII.

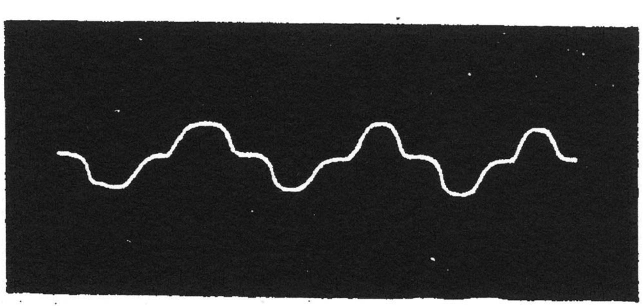

Fig. 8. Sketch of theoretical water-hammer pressure at midpoint of tube (Kries 1892)

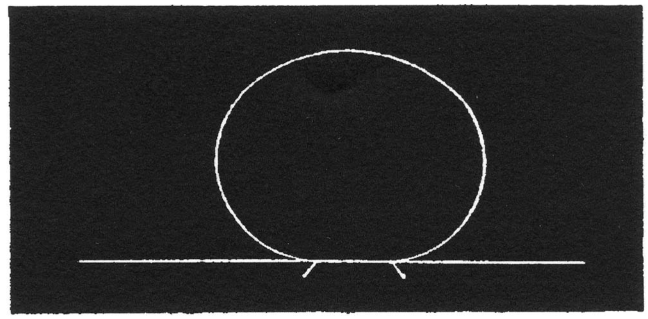

Fig. 9. Sketch of tube flattening (Kries 1892)

Conclusion

In 1883, Johannes von Kries published the theory of water hammer in a study of blood flow in arteries. He derived the Joukowsky formula before Joukowsky and Frizell in 1898. He considered skin friction in unsteady laminar flow and thus derived formulas for wave attenuation and line pack. The theory was con-

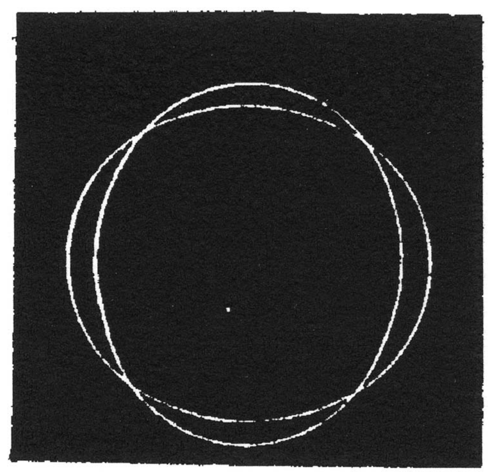

Fig. 10. Sketch of tube ovalizing (Kries 1892)

firmed by experimental results obtained in water-filled rubber hoses. In 1892, he published the first textbook describing "classical" water hammer. It presents formulas for phase-velocity and damping that are frequency-dependent because of skin friction; and in this sense, it is the first contribution to the—these days popular—subject of unsteady friction (Zielke 1968; Bergant et al. 2001; Vardy and Brown 2003, 2004; Ghidaoui et al. 2005).

This paper gives the work of Kries a place in the history of transients in fluids and solids. It also provides the first synopsis in English of his investigations on water hammer, thus making it accessible to a wider readership.

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Notation

The following symbols are used in this paper:

A = cross-sectional area (m^2);
c = sonic wave speed (m/s);
$c(\omega)$ = phase velocity (m/s);
D = internal tube diameter (m);
E = Young modulus (Pa);
e = tube wall thickness (m);
F = force (N);
K = fluid bulk modulus (Pa)
K^* = effective fluid bulk modulus (Pa);
L = pipe length (m);
p = fluid pressure (Pa);
t = time (s);
v = velocity (m/s);
Δ = change, jump;
ε = strain;
μ = dynamic viscosity (Pa s);
ν = kinematic viscosity (m^2/s);
ρ = mass density (kg/m^3);
σ = stress (Pa); and
ω = circular frequency (rad/s).

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