Abstract
This article is intended for engineers who are not familiar with waterhammer, and for those who are familiar with the phenomenon but unaware of the fact that transient pressures can exceed the classical Joukowsky value. Waterhammer and the crucial role of the speed of sound are presented. Additional complications, mitigation measures and beneficial use are addressed. Waterhammer is a practical problem and will always be so.

The Joukowsky formula
Pumps start and stop, and valves open and close. When these actions are abrupt, severe pressure waves are generated which travel at high speed through the entire piping system and which may cause damage to machinery and supports. This phenomenon is called waterhammer, or alternatively fluid transients or pressure surges. Figure 1 is used to explain the basic mechanisms. When a liquid that flows steadily at velocity \( V_0 \) is suddenly stopped by closing a valve (or switching-off a pump), the upstream pressure rises and the downstream pressure drops. The generated pressure rise \( \Delta P \) travels upstream (at the speed of sound \( c \)) to convey the message of a closed valve. The pressure rise reflects negatively at the reservoir and the returning wave causes a pressure drop at the valve when it arrives there. The same thing happens (with opposite sign and different timing) to the pressure drop \(-\Delta P\) at the downstream side. The final result is a fluctuating pressure at both sides of the valve, at first instant causing a force of \( pD^2\Delta P/2 \) acting on the valve, where \( D \) is the diameter of the pipe. The pressure fluctuation gives an (often audible) hammering effect. The period of the upstream fluctuation is \( T = (4L_u)/c_u \) where \( L_u \) is the upstream length and \( c_u \) the upstream wave speed. The amplitude of this fluctuation is obtained from

\[
\Delta P = -\rho c \Delta V \quad \text{or} \\
\Delta H = -\frac{c}{g} \Delta V 
\]

(1)

where \( \rho \) is the mass density of the liquid, \( g \) is the acceleration due to gravity, \( H \) is the pressure-head, and \( \Delta V \) is the change in velocity, which is \(-V_0\) here. Formula (1) represents the well-known Joukowsky pressure. In steel pipes carrying water, a velocity change of 1 m/s will result in Joukowsky pressures of the order of 10 bar.

This gives rise to large but short-lived pressure fluctuations which may be detrimental to the system. The subject has been extensively dealt with in textbooks [e.g. 1a-e] and reviews [e.g. 2a-d]. A little bit of history can be found in the articles [3a-d]
Pressure Wave Speed

Isothermal fluid flows in pipes are described by pressures $P$ and convective velocities $V$. Waterhammer is an acoustic phenomenon where the speed of sound $c$ is a key player.

The acoustic pressure $P = \rho c V$ stands midway between the hydrodynamic pressure $P = \rho V^2$ and the thermodynamic (gas) pressure $P = \rho c^2$, where $\rho$ is the mass density of the fluid. The acoustic velocity $c$ in liquid-filled pipes is given by

$$\frac{1}{c^2} = \frac{1}{c_0^2} + \frac{1}{c_1^2} + \frac{1}{c_2^2} \quad \text{with} \quad c_0 = \sqrt{\frac{K}{\rho}}, \quad c_1 = \sqrt{\frac{E e}{\rho D}}, \quad c_2 = \sqrt{\frac{P - P_v}{\rho \alpha}},$$

where $K$ and $E$ are the elasticity of the liquid and the pipe wall, respectively; $D$ and $e$ are the pipe's inner diameter and wall thickness, respectively; $P_v$ is the liquid's vapour pressure and $\alpha$ ($0 < \alpha < 1$) is the volume fraction of free gas. The velocity $c_0$ is the speed of sound in an unconfined liquid (say the ocean), the velocity $c_1$ that of wave propagation in a very flexible hose (or blood vessel), and the velocity $c_2$ accounts for the slowing-down effect of free gas (e.g. distributed air bubbles in water). There are no contributions of $c_0$, $c_1$ and $c_2$ for the limit cases $K = \infty$ (incompressible liquid), $E = \infty$ (entirely rigid pipe wall) and $\alpha = 0$ (no free air), respectively. The wave speed $c$ vanishes in regions of vaporous cavitation where $P = P_v$. The wave speed $c$ determines the magnitude of the pressure rise according to the Joukowsky formula (1). Typical values for steel pipes carrying water are: $\rho = 1000 \text{ kg/m}^3$, $K = 2 \text{ GPa}$, $E = 200 \text{ GPa}$, $D/e = 40$, $P_v = 2 \text{ kPa}$ and $\alpha = 10^{-7}$. The wave speed for these values according to formula (2) is:

c = 1195 \text{ m/s}, \quad \text{with} \quad c_0 = 1414 \text{ m/s},
\quad c_1 = 2236 \text{ m/s} \quad \text{and} \quad c_2 = 44497 \text{ m/s}

for $P = 200 \text{ kPa}$. Figure 2 depicts the wave speed's dependence on the free air content.

Exceeding Joukowsky

The Joukowsky formula (1) is valid for pure liquid flow in uniform pipelines without branches, where the valve is structurally fixed and its closure takes less time than the pressure wave return period $2L/c$. In general, “Joukowsky” is a very good estimate of the first – and often largest – pressure rise in the system. However, pressures exceeding “Joukowsky” may occur later on in at least three instances: 1) an air pocket is entrapped somewhere along the line, 2) the pressure drops to vapour pressure somewhere along the line, so that vapour bubbles form (cavitation and column separation), and 3) the valve moves under the high waterhammer load (fluid-structure interaction).

These three instances are illustrated further. Visco-elastic wall behaviour (typical of plastic pipes) does not lead to pressures higher than “Joukowsky”.

Fig. 1: Hydraulic grade line and pressure waves caused by instantaneous valve closure

Fig. 2: Wave speed $c$ as function of free-gas fraction $\alpha$ for three different absolute pressures
Pipeline
The test problem and its numerical solutions are taken from Refs [4a-b]. A copper pipe of length \( L = 37.2 \text{m} \), diameter \( D = 22 \text{mm} \), and wall thickness \( e = 1.6 \text{ mm} \), connects two tanks filled with pressurised water (Figure 3). Waterhammer is initiated by rapid (within 10 ms) valve closure. The initial velocity \( V_0 \) is either 0.2 m/s or 0.3 m/s, and the constant pressure head in the upstream tank is 32 m. The wave speed \( c \) is 1319 m/s.

Air Pocket
Uniformly distributed free air reduces the wave speed \( c \) [through \( c^2 \) in formula (2)] and will not give rise to pressures exceeding “Joukowsky”. The situation is different when an individual air pocket is entrapped somewhere along the line. In the next example a single gas pocket with a volume of 5 cm³ (void fraction \( \alpha_{0.3/8} = 0.01 \) in the numerical model [4b]) is trapped at the 3/8th point (i.e. 14 m) from the valve in Fig. 3. The calculated pressure-head history at the valve is compared with the classical case (without air pocket) in Figure 4. The whimsical pressure trace shows positive and negative pressure peaks exceeding “Joukowsky” up to 15%. Air pockets make things unpredictable, not only because they cause strange nonlinear system behaviour, but also because in the real world their locations and sizes are not known in advance.

Column Separation
Waterhammer not only leads to high pressures but also to low pressures. Sub-atmospheric pressures involve the danger of pipe collapse, especially in the case of buried pipelines. When the pressure comes below a certain level, cavitation occurs. Distinction should be made between gaseous and vaporous cavitation [2b]. Gaseous cavitation occurs when the pressure falls below the saturation pressure of the gas, so that it comes out of solution. This is a relatively slow process compared to vaporous cavitation. Vaporous cavitation occurs when the pressure drops to the vapour pressure and cavities will form in the liquid. When the vapour cavities appear as tiny bubbles dispersed throughout the liquid along great lengths of pipe, it will be referred to as distributed cavitation or flashing flow (void fraction \( \alpha \approx 0 \)).

When the vapour cavities coalesce and form one local bubble occupying a large part of pipe cross-section, it will be referred to as column separation (\( \alpha \approx 1 \)). Column separations generally occur near specific points in a pipe system such as valves, pumps, bends and high points. Column separations occasionally occur in pipe intermediate points when two low-pressure waves meet.
The collapse of column separations is usually attended with almost instantaneous pressure rises. These may be avoided by positioning air inlet valves at critical points in the pipe system. In general the policy is to prevent cavitation.

Figure 5 shows a conceivable pressure-head history for waterhammer with column-separation. The minimum pressure-head predicted by the classical model (without column-separation) is well below the vapour pressure-head, which is about 0.2 m for water at room temperature. This means that in reality the liquid starts to vaporise, whereby a large cavity is formed at the valve.

The vapour bubbles keep the local pressure at a flat bottom level. Sometime after the collapse of the first cavity, the overlap of distinct positive pressure waves traveling in the pipeline cause a large peak of short duration, such that “Joukowsky” is exceeded by more than 60%.

**Fluid-Structure Interaction**

In the classical theory of waterhammer the hoop elasticity of the pipe is incorporated in the wave speed $c$ [through $c_1$ in formula (2)]. Pipe inertia and pipe motion are not taken into account. This is acceptable for rigidly anchored pipe systems. For less restrained systems (Figure 6) fluid-structure interaction may become of importance. In that case waterhammer and pipe vibration should be treated simultaneously.

Two fundamental liquid-pipe interaction mechanisms are Poisson coupling and junction coupling [2c-d]. Poisson coupling relates the pressures in the liquid to the axial stresses in the pipe wall through the radial contraction or expansion of the pipe. It is named after the Poisson contraction in the theory of elasticity and associated with the breathing or hoop mode of the pipe. Poisson coupling leads to precursor waves. These are stress-wave induced disturbances in the liquid which travel faster than, and hence in front of, the classical waterhammer waves.

Poisson coupling acts along the entire pipe, whereas junction coupling acts at specific points such as unrestrained closed ends, pipe bends and branches. A standard example is the moving elbow, which induces pressure waves in the liquid through a combined pumping (compressing) and storage (decompressing) action.
The pipeline in Fig. 3 is now assumed not to be restrained against axial motion along its entire length and the massless valve is free to move, while the upstream tank is structurally fixed.

The pressure rise generated by its fast closure pushes the valve in the downstream direction, thereby creating additional storage for the liquid and as a result a lower initial pressure rise, as can be seen from Figure 7, which shows the combined effects of Poisson and junction coupling.

The liquid is not brought entirely to rest: it has the velocity of the moving valve, which via formula (1) can be deduced from the calculated pressure rise at time zero.

It is evident that fluid-structure interaction, when compared with the classical model, causes larger extreme pressures, high-frequency fluctuations, and a small phase shift. “Joukowsky” is peak-wise exceeded up to 80%.

Surge Protection Methods and Beneficial Use of Waterhammer

In order to prevent damage, waterhammer can be suppressed and controlled by devices like: surge tanks and towers (Figure 8), air chambers, air-inlet valves, pressure relief valves, rupture disks, pump flywheels and flexible hoses. Another traditional method of surge protection is valve stroking, where the valve closure is optimised to give a minimal pressure rise. Check valves are usually damped for this reason. In practice waterhammer simulations are carried out to judge whether the quite expensive devices are necessary and if so, what their locations and dimensions should be. Predicted maximum pressures prescribe the required strengths of the pipes, joints and supports.
We like to finish this article with some positive news: pressure waves can also be used beneficially. The oldest and most striking example is the hydraulic ram pump (Figure 9).

Hydraulic rams are self-operating pumps which harness the waterhammer phenomenon by amplifying an available small pressure-head by a factor of twenty to forty [5a-gl].

They need no maintenance and are used in rural areas where no electricity or fuel is available. Pressure waves are also extensively utilised in the non-invasive detection of leaks and blockages in pipelines. Industrial cleaning processes occasionally exploit the phenomenon of waterhammer.

**Conclusion**

Waterhammer has been known for centuries and will continue to be known for centuries, simply because incidents and accidents seem to be unavoidable. However, a good awareness of the phenomenon by operators, designers, hydraulic engineers, and the like, will reduce the number of serious waterhammer events.

We have given a simple and hopefully clear introduction to the subject. The interested reader is referred to the list of references below.

**References**


**Reviews:**


**History:**


**Numerical Simulations:**


**Hydraulic Ram Pumps:**


