SOME INTRIGUING ASPECTS OF BOUNDARY CONDITIONS IN WATER HAMMER

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ABSTRACT

The influence of boundary conditions on water hammer in a traditional reservoir-pipeline-valve system is studied. At first sight this seems a trivial matter. However, a second look reveals some surprising aspects not known to many and therefore presented herein. Water hammer consists of transient pressures P and transient velocities V, the histories of which are largely determined by sonic speed c, pipeline length L, liquid density ρ, initial flow velocity V_0, and effective valve closure time τ. The last parameter is zero herein, because, for clarity, we consider instantaneous valve closures in frictionless pipelines. The questions raised and discussed in this paper are: i) $4L/c$ versus $2L/c$ systems, and do $3L/c$ systems exist?; ii) is an orifice an open end or a closed end, or neither of these?; iii) can all the hydraulic system's resonance frequencies be found from a measured water-hammer history?; iv) can measured pressures reliably be used as boundary conditions?; v) why can we have two boundary conditions at one end in the frequency domain, and not so in the time domain?; vi) are wave speed and phase velocity the same thing?; vii) is my valve structurally fixed? Things are explained from basic principles and simple test cases.

Keywords: water hammer, pressure surges, fluid transients

1 INTRODUCTION

Water hammer is a well-known and well-studied phenomenon that deserves continuing attention simply because it will always be there with incidents and accidents. It is assumed herein that the reader is fully familiar with the phenomenon, the Joukowsky pressure, method of characteristics (MOC), transfer matrices (TMM), etc., as documented in classic texts like those of Wylie and Streeter (1993), Ghidaoui et al. (2005) and Chaudhry (2014). Seven simple questions are posed in an attempt to find definite answers. To keep things as basic as possible, a frictionless pipeline is considered with either valves or reservoirs at its extremities. The purpose of the paper is to show the remarkable influence of boundary conditions on hydraulic system behaviour.

2 4*L*/*C* VERSUS 2*L*/*C* SYSTEMS, AND DO 3*L*/*C* SYSTEMS EXIST?

A reservoir-pipe-valve (RPV) system is known as a $4L/c$ system because its period of water-hammer oscillation is $4L/c$ when the valve is fully closed (Fig. 1a). A system with either two open or two closed ends is a $2L/c$ system (Fig. 1c). There are different ways to see why this is the case. First, one can follow the wave fronts and the reflections at the ends to see that things repeat after a certain period T. In an RPV system the reflections are different at each end and this fact results in a two times smaller period than when the reflections are the same. Second, one can find the fundamental period of free oscillation from an elementary theoretical analysis. Third, one can excite the system (harmonically or by impact) and observe at which frequency $f = 1/T$ resonance occurs. But, do $3L/c$ systems exist?

3 IS AN ORIFICE AN OPEN END OR A CLOSED END, OR NEITHER OF THESE?

The transient behaviour with pipe ends that are fully open or fully closed is clear. But what happens when we have something in between, say a partially closed valve or an orifice? Do we still have our traditional $4L/c$ or $2L/c$ systems, or do we have something in between, say a $3L/c$ system? The answer to this question can be found in (Wylie and Streeter, 1993, Section 12-5), in terms of complex analysis, and by Peng and Moody (2003), who applied Lapace transforms and infinite series. The main message is that the orifice acoustically acts as a closed end as long as its constant impedance (resistance) is larger than the impedance ρc of the liquid, and that it acts as an open end when its impedance is smaller. This is a little strange, because it means that hydraulic systems suddenly change from 4*L*/*c* systems to 2*L*/*c* systems when valves are opened gradually. This unexpected discontinuity has been investigated by Tijsseling et al. (2012) who found out that the change of acoustic behaviour yet is gradual. The change of fundamental period depends on the orifice's acoustic impedance and at the critical point the orifice acts as a non-reflecting boundary condition thereby prohibiting standing oscillations. This might seem a little strange, but stranger things happen when a closed end houses a trapped air pocket behaving as a massless spring (Tijsseling et al., 1999). The stiffness of the entrapped gas pocket may range from very soft to very hard and that makes the system range from 2*L*/*c* to 4*L*/*c*. Now there is *not* a critical point where the gas pocket acts as a non-reflecting boundary, because its impedance depends on the period of oscillation. At last it is noted that orifice resistance and skin friction make the acoustic system nonlinear, so that there is not a *constant* period of oscillation.

It is commonly assumed that a system with an upstream reservoir will behave in a 4*L*/*c* manner if the downstream boundary is a prescribed-pressure variation (as with P = constant, for instance), but in a 2*L*/*c* manner if the downstream boundary is a prescribed-velocity variation (as with V = constant, for instance). But what happens if the prescribed-pressure variation is chosen to be identical to the familiar 4*L*/*c* signal shown in Fig. 1a (where the initial velocity $V_0 > 0$)? The velocity at the valve will necessarily become zero and remain zero (Fig. 1b), and so the system is 4*L*/*c*, not 2*L*/*c* as implied by the general statement at the beginning of this paragraph. At the other extreme, if a 2*L*/*c* pressure signal is imposed at the downstream end (Fig. 1c), resonance occurs at 2*L*/*c* as expected (Fig. 1d) because the system is equivalent to an open-open pipeline. Strangely enough, resonance (or a beat of very low frequency) also occurs for a 3*L*/*c* pressure excitation (Fig. 1e) as is evident from Fig. 1f, although the excitation period 3*L*/*c* is larger than the fundamental period 2*L*/*c*. The response "period" *L*/*c* is the result of a combined free and of forced oscillation.

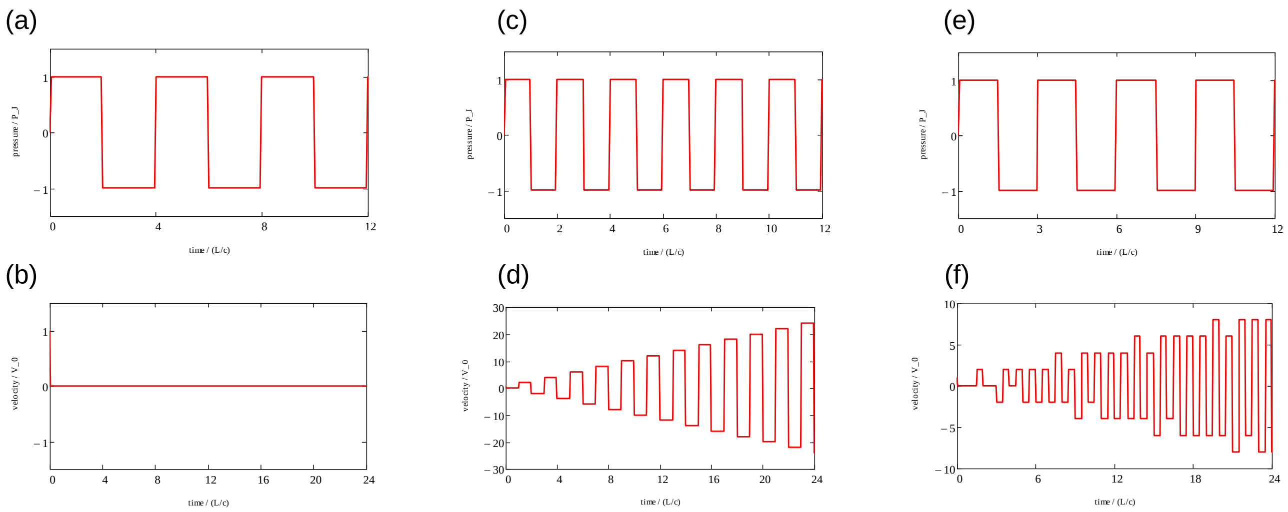

Figure 1. Imposed end pressures (divided by the Joukowsky pressure) and same-end velocity responses (divided by the initial velocity) versus time (divided by *L*/*c*): (a-b): 4*L*/*c* excitation, (c-d): 2*L*/*c* excitation, (e-f): 3*L*/*c* excitation. Exact calculations according to (Tijsseling and Bergant, 2007).

4 CAN ALL THE HYDRAULIC SYSTEM'S RESONANCE FREQUENCIES BE FOUND FROM A MEASURED WATER-HAMMER HISTORY?

The resonance frequencies of a system can be detected from the response to an impact load (Zhang et al., 1999). If the impact load misses "frequency content", not all potential resonances will be activated. Suppose that one uses water hammer to excite an RPV system. In this case, the response ideally contains all relevant system information. Suppose, however, that a short pulse is generated by closing and then re-opening a valve. In this case, the system will be 4*L*/*c* for part of the time and 2*L*/*c* for another part of the time. This may lead to erroneous conclusions. Not re-opening of the valve gives the frequency response of a 4*L*/*c* system and not that of the original 2*L*/*c* system.

5 CAN MEASURED PRESSURES RELIABLY BE USED AS BOUNDARY CONDITIONS?

This is a common procedure that possibly is not valid when the system is operating near resonance (Tijsseling et al., 2010). It is noted that (unsteady) friction, (structural) damping and fluid-structure interaction (FSI) become important in a resonating system.

6 WHY CAN WE HAVE TWO BOUNDARY CONDITIONS AT ONE END IN THE FREQUENCY DOMAIN, AND NOT SO IN THE TIME DOMAIN?

Water-hammer analyses can be carried out either in the time domain or in the frequency domain. The time domain is more suitable for impact excitations like instantaneous valve closures, whereas the frequency domain is the better choice for oscillatory excitation and resonance. Nevertheless, the analyses in both domains concern and contain exactly the same information. It is therefore strange that in the time domain one boundary condition is required at each pipe end, because (for sub-sonic systems) there is only one information-carrying characteristic line reaching the boundary (Fig. 2a), whereas in the frequency domain two boundary conditions is also a valid possibility. One way to explain this is as follows. The time-domain variables P and V are transformed to the frequency-domain variables $\tilde{P}$ and $\tilde{V}$, for example by a Laplace transform. The latter is a weighted average over time, where time goes from zero to infinity. But this means that the solution for P and V is *assumed* to be known for all times t in advance in order to determine $\tilde{P}$ and $\tilde{V}$, and therefore it is allowed to travel backwards in time as shown in Fig. 2b, where two information-carrying characteristic lines meet at the boundary. This boundary can do without extra condition, but then – to compensate – the other one needs two.

One important restriction for frequency-domain analysis is that the system must be linear or linearised, and therefore does not contain exactly the same information as the time-domain system. The frequency domain is inappropriate for strongly non-linear systems, i.e. equations *and* boundary conditions. Frequency-domain calculations are usually performed with complex numbers in order to take full advantage of the pleasant properties of exponential functions; they may lead to exact solutions, where the time-domain calculation does not.

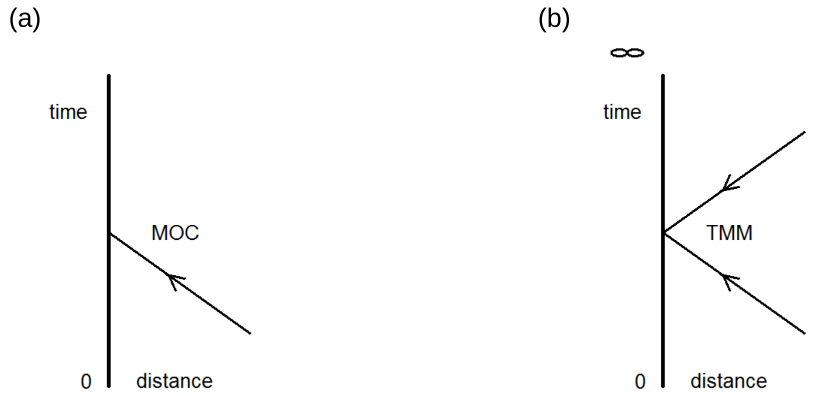

Figure 2. Characteristic lines in the distance time plane: (a) MOC, (b) TMM.

7 ARE WAVE SPEED AND PHASE VELOCITY THE SAME THING?

The wave speed in real systems is usually found from the fundamental water-hammer period T under the assumption that the pipeline is a $4L/c$ or a $2L/c$ system. We have seen that a partially open (or leaking) valve does not affect this assumption, but that an entrapped air pocket at a dead end might do so. For short pipes it is noticed that waves reflect from a point somewhat into the reservoir, so that L should be taken a little longer than the actual pipe length. Another issue discussed by Tijsseling and Vardy (2015) is that a travelling wave front in the time domain is something different from a sinusoidal wave train in the frequency domain. The wave front is a local jump, whereas the wave train is global in the sense that it covers the entire pipeline. The front travels with the wave speed and the train has a phase velocity. For non-dispersive systems these are the same, but for dispersive systems the latter is frequency-dependent and may have magnitudes either smaller or larger than the physically possible maximum wave speed, say 1480 m/s in pure water. In a similar way, time delays at boundaries (e.g. the response of a pump to a sudden change in flow rate) cause diffusion of reflected signals and hence change the frequency-response character of the overall flow.

8 IS MY VALVE STRUCTURALLY FIXED?

The valve in an RPV system experiences heavy and sudden loads during a water-hammer event, in particular when repeated column-separations occur (Bergant et al., 2006). The valve will – depending on its anchorage – certainly move or vibrate to a certain extent. Does this motion or vibration have an influence on the system's $4L/c$ or $2L/c$ behaviour? The importance of this fluid-structure interaction mechanism depends on the relevant time scales (Wylie and Streeter, 1993, Section 11-2; Tijsseling and Vardy, 2008). Wylie and Streeter (1993, Section 6-8) considered a simple spring-mass model where the valve has mass m and stiffness k. If the valve has no explicit support, the spring stiffness comes from the adjacent pipe section so that $k = EA/L$, where E and A are Young's modulus and cross-sectional area of the pipe wall. This model addition will certainly affect system behaviour, just like the gas pocket modelled as a massless spring (but now with significant mass added to it). If the spring is relatively soft, the liquid will tend to behave similarly to a rigid column for which $L/c = 0$. If the spring is extremely stiff or the valve mass very large, we have a closed end.

9 CONCLUDING REMARKS

A mix of issues related to boundary conditions in water hammer has been addressed without presenting the mathematics behind it. The phenomena have been explained as simply as reasonably possible and in this sense the paper has an educational character. Hopefully the reader has been enthused to study some of the discussed aspects of hydraulic transients and in particular the questions that – in his/her opinion – have not been answered satisfactorily.

REFERENCES

Bergant A., Simpson A.R. & Tijsseling A.S. (2006) Water hammer with column separation: A historical review. Journal of Fluids and Structures, Vol. 22, No. 2, pp. 135-171.

Chaudhry M.H. (2014) *Applied Hydraulic Transients (third edition)*. Springer.

Ghidaoui M.S., Zhao M., McInnis D.A. & Axworthy D.H. (2005) A review of water hammer theory and practice. Applied Mechanics Reviews, Vol. 58, No. 1, pp. 49-76.

Peng M.M. & Moody F.J. (2003) Natural frequencies in pipes with orifice terminations. Proc. of the ASME Int. Mechanical Engineering Congress, Washington DC, USA, November 2003, Paper IMECE2003-55039; ASME-FED 259, pp. 165-169.

Tijsseling A.S. & Bergant A. (2007) Meshless computation of water hammer. Proc. of the 2nd IAHR Int. Meeting of the Workgroup on Cavitation and Dynamic Problems in Hydraulic Machinery and Systems, Timişoara, Romania, October 2007, Scientific Bulletin of the "Politehnica" University of Timişoara, Transactions on Mechanics, Vol. 52(66), No. 6, pp. 65-76.

Tijsseling A.S., Hou Q., Svingen B. & Bergant A. (2010) Acoustic resonance in a reservoir-pipeline-orifice system. Proc. of the ASME 2010 Pressure Vessels and Piping Division Conf., Bellevue, Washington, USA, July 2010, Paper PVP2010-25083.

Tijsseling A.S., Hou Q., Svingen B. & Bergant A. (2012) Acoustic resonance in a reservoir - double pipe - orifice system. Proc. of the ASME 2012 Pressure Vessels and Piping Division Conf., Toronto, Canada, July 2012, Paper PVP2012-78085.

Tijsseling A.S., Kruisbrink A.C.H. & Pereira da Silva A. (1999) The reduction of pressure wavespeeds by internal rectangular tubes. Proc. of the 3rd ASME & JSME Joint Fluids Engineering Conf., Symposium S-290 Water Hammer (Editor J.C.P. Liou), San Francisco, USA, July 1999, Paper FEDSM99-6903, ASME-FED 248, pp. 1-8.

Tijsseling A.S. & Vardy A.E. (2008) Time scales and FSI in oscillatory liquid-filled pipe flow. BHR Group, Proc. of the 10th Int. Conf. on Pressure Surges, Edinburgh, United Kingdom, May 2008, pp. 553-568.

Tijsseling A.S. & Vardy A.E. (2015) What is wave speed? BHR Group, Proc. of the 12th Int. Conf. on Pressure Surges, Dublin, Ireland, November 2015, pp. 343-360.

Wylie E.B., Streeter V.L. & Suo L. (1993) *Fluid Transients in Systems*. Prentice Hall.

Zhang L., Tijsseling A.S. & Vardy A.E. (1999) FSI analysis of liquid-filled pipes. Journal of Sound and Vibration, Vol. 224, No. 1, pp. 69-99.