Weighted Model Counting on the GPU by Exploiting Small Treewidth

Johannes K. Fichte[1] Markus Hecher[2,3] Stefan Woltran[2] Markus Zisser[2]

[1] TU Dresden, Germany
[2] TU Wien, Austria
[3] University of Potsdam, Germany

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Shonan, Japan

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Motivation

Model Counting (#SAT)

- Generalizes Boolean satisfiability problem (SAT)
- #SAT: output the number of satisfying assignments
- Various applications in AI and reasoning, e.g.,
  - Bayesian reasoning [Sang et al.'05]
  - Learning preference distributions [Choi et al.'15]
  - Infrastructure reliability [Meel et al.17]
- Computational complexity: #P-hard [Roth'96]

Problem of Interest

SAT-Problem (Boolean Satisfiability Problem)

Given: Propositional formula F.
Question: Is there a truth assignment τ to the variables in F such that F_τ evaluates to 1 (satisfiable).

Skip Example

Input normal form

- Conjunctive normal form (CNF)
- Form: $F = (\ell_1 \vee \ell_2 \vee \ell_3) \wedge \ldots \wedge (\ldots)$ where ℓ_i either x or $\neg x$

#SAT (Number SAT)

- Number of of satisfying truth assignments to F.

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$$(a \vee \neg b \vee d) \wedge (\neg a \vee \neg c \vee \neg d) \wedge$$

$$(\neg a \vee \neg d) \wedge (b \vee c) \wedge (d \vee e) \wedge (\neg b \vee \neg e)$$

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$\Rightarrow$ Satisfiable

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Problem of Interest cont.

Weighted Model Counting (WMC)

- Generalizes #SAT
- Given Boolean formula, e.g., $F = (x \vee y) \wedge (\neg x \vee \neg y)$

  Function that maps literals to reals between 0 and 1, e.g.,

  $x \mapsto 0.4$, $\neg x \mapsto 0.6$, $y \mapsto 0.7$, $\neg y \mapsto 0.3$
- Weight of an assignment α is the product over the weights of its literals, i.e., $w(\alpha) := \Pi_{v \in \alpha^{-1}(1)} w(v) \cdot \Pi_{v \in \alpha^{-1}(0)} w(\neg v)$,

  e.g., $\alpha(x) = 1$, $\alpha(y) = 0 \Rightarrow w(\alpha) = 0.4 \cdot 0.3 = 0.16$
- Weighted model count (WMC) of formula is the sum of weights over all its satisfying assignments, e.g.,

  $w(F) = w(\alpha_1) + w(\alpha_2) = 0.12 + 0.42 = 0.54$

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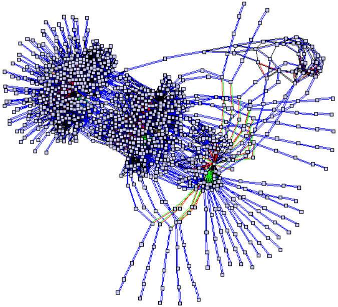
Theory:

SAT cannot be solved faster than $2^{o(n)}$ steps! (ETH)

Idea:

- Practical instances are usually highly structured
- Structure can be exploited by algorithms



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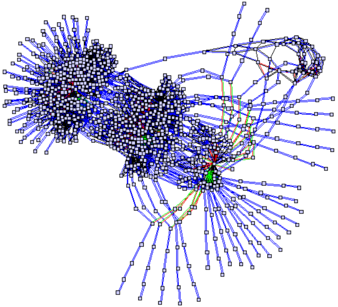

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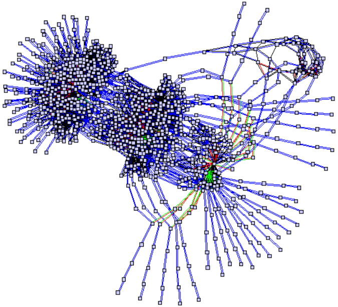

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Motivation: A somewhat different approach.

#SAT/WMC Solving

- There are already various solvers based on various techniques: approximate (Meel) / CDCL (Baccus/Thurley) / knowledge compilation based (Darwiche et al.)

Parameterized Algorithms

- Lots a theoretical work over last 20 years and various algorithms for #SAT

Research Question

Are (theoretical) algorithms from parameterized complexity even useful for implementations in #SAT/WMC solving?

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Topic of the Talk

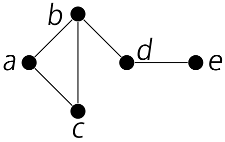
Solve #SAT/WMC by means of an implementation of a parameterized algorithm that exploits small treewidth.

Tree Decompositions

Treewidth ‣ Definition & Example

- Most prominent graph invariant
- Small treewidth indicates tree-likeness and sparsity
- Can be used to solve #SAT/WMC by defining graph representations of the input formula

Tree Decompositions

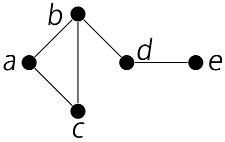


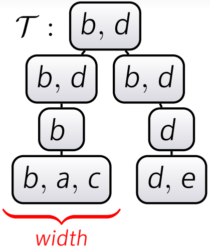
Treewidth

Definition & Example

- Treewidth defined in terms of tree decompositions (TD)
- TD: arrangement of graph into a tree + bags s.t. ...
- Treewidth: width of a TD of smallest width

Tree Decompositions





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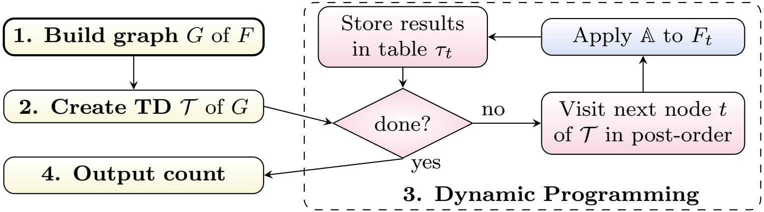

Part:

A) Background & Basic Concepts

Treewidth, Graph Representation (1) + Dynamic Programming (3) [Samer & Szeider JDA'10]

B) Finding TDs (2)

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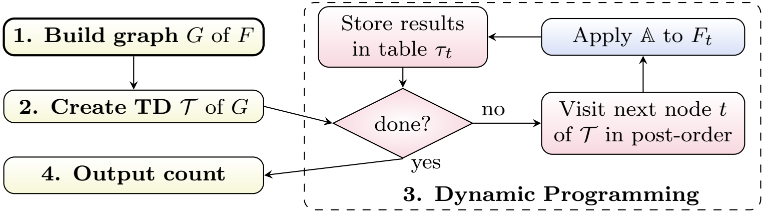

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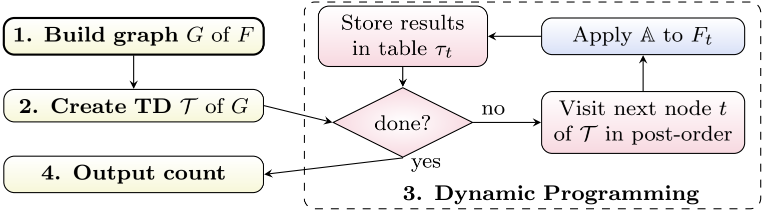

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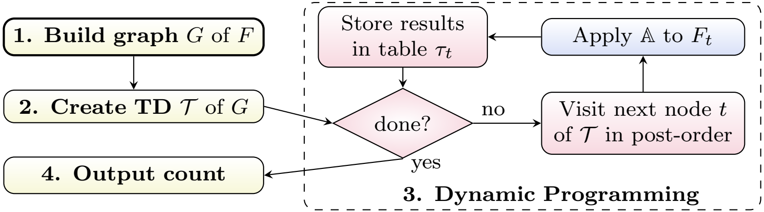

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Works well even for relatively large instances.

Thanks to the Parameterized Algorithms and Computational Experiments Challenge (PACE) ’16/’17!!!

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How to “use” tree decompositions for #SAT/WMC?

By Example

$v_a \vee v_b.\ v_a \vee v_c.$
$v_b \vee v_c.\ v_b \vee v_d.$
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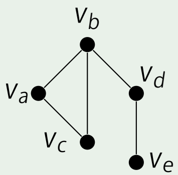
Formula F



Primal graph

Graph Representations

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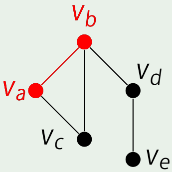
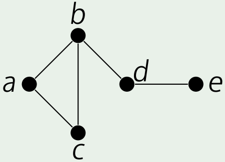


Primal graph

Exploiting Tree Decompositions (TDs)

Dynamic Programming for SAT [Samer & Szeider'10]



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2. Algorithm for SAT
3. Combine solutions

Runtime: $\mathcal{O}(2^k \cdot \|F\|^2)$ where F is the input formula and k the width of TD

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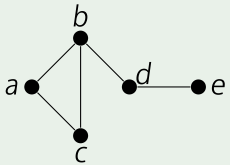

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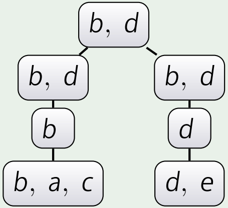

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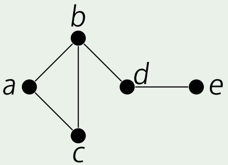

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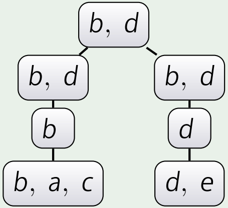

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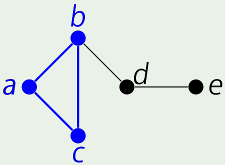

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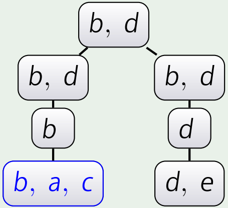

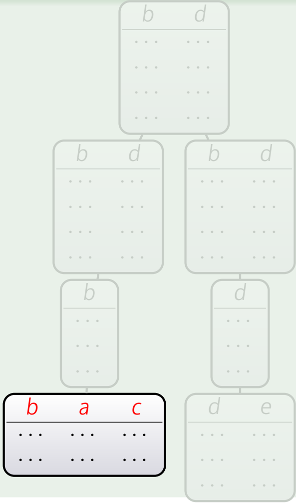

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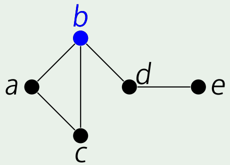

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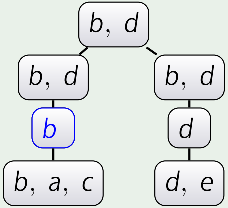

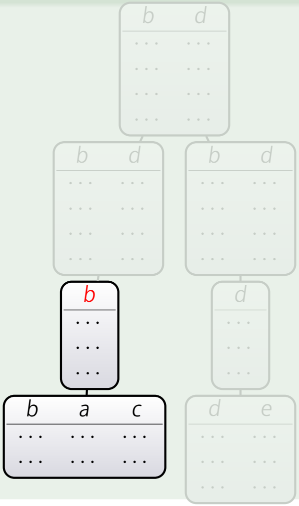

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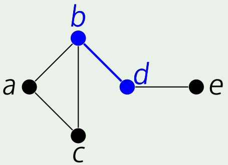

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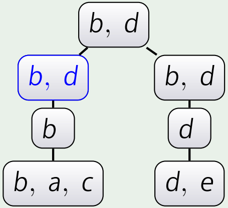

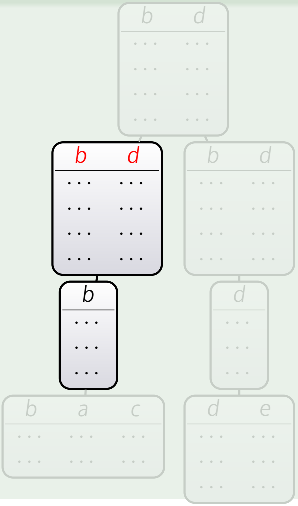

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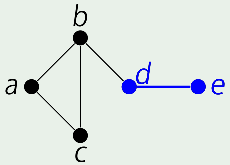

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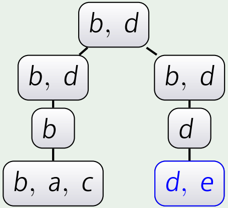

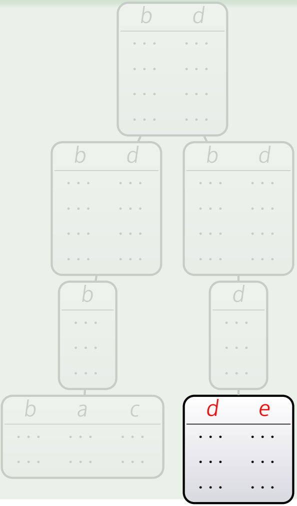

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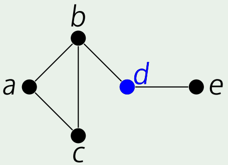

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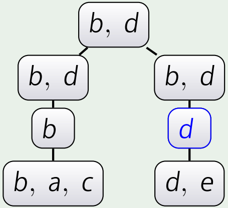

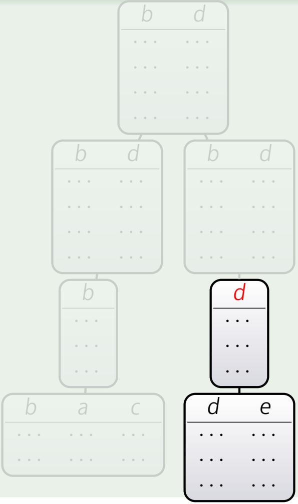

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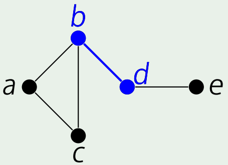

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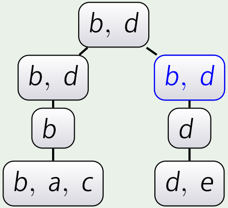

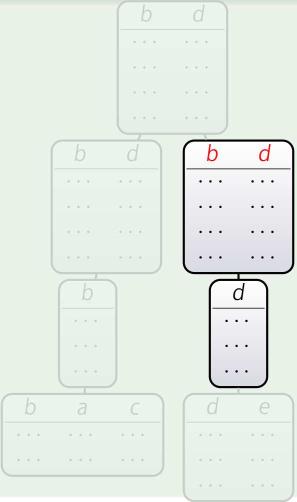

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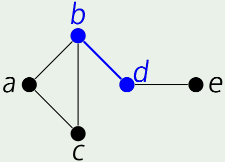
Dynamic Programming for SAT [Samer & Szeider'10]



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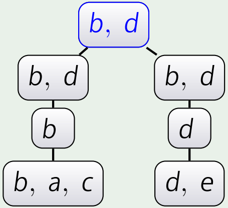

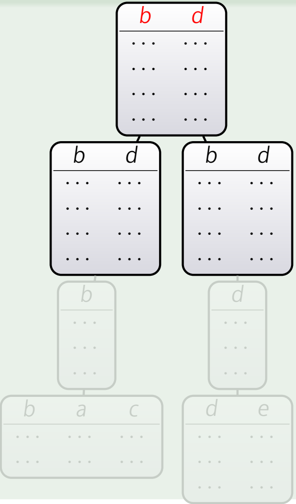

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Exploiting Tree Decompositions (TDs)

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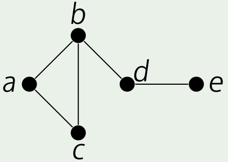

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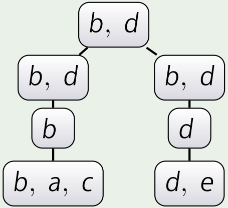

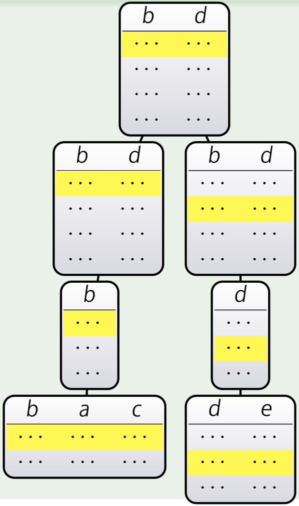

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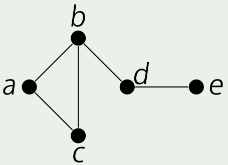

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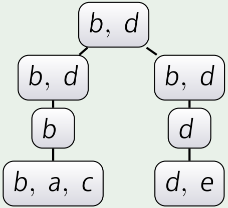

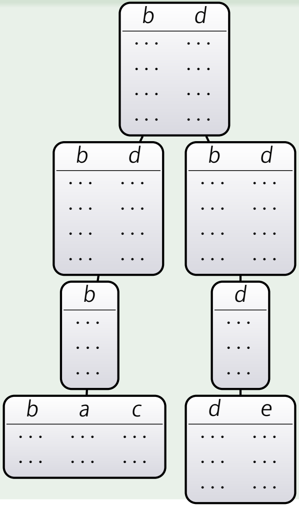

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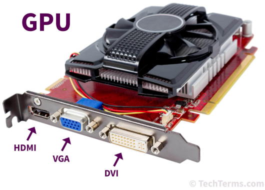
A GPU-based #SAT/WMC-solver

OR how to go parallel?



Dynamic Programming on the GPU

How to parallelize DP?

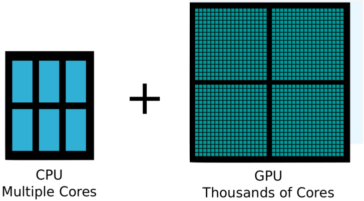
1. Compute tables for multiple nodes in parallel

⇒ Does not allow for immediate massive parallelization due to dependencies to children

2. Distribute computation of rows among different computation units

⇒ Allows with right hindsight for massive parallelization

Why: computation of rows are independent



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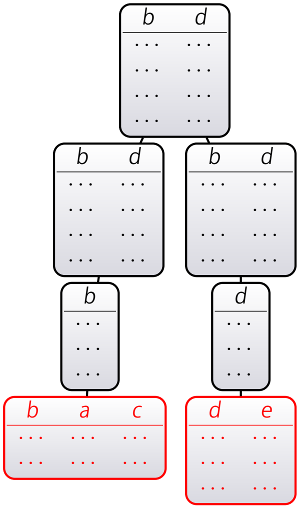

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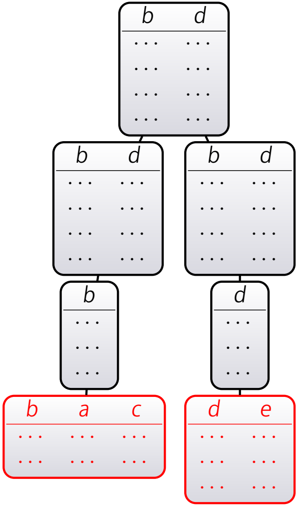

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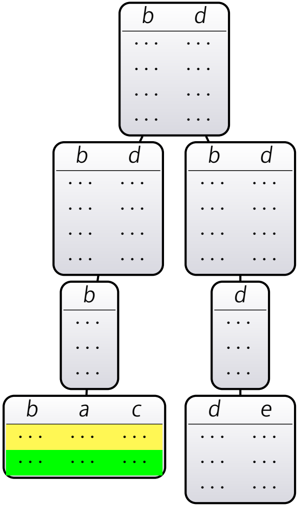

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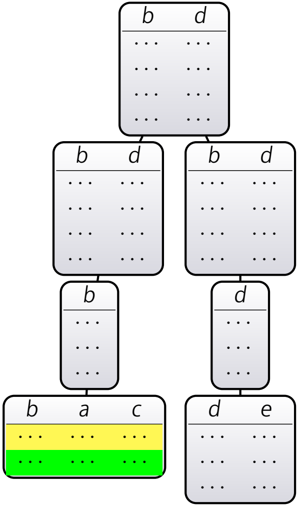


Implementation (gpuSAT1)

Right hindsight?

- Data structures: a "pixel" represents #solutions (store data as array)
- Table merging: merge small bags (< 14)
- Table splitting: split large tables
- Weight: avoid double overflows by factor for #SAT
  (so it's actually WMC with uniform weights).

Implementation

- OpenCL: vendor and hardware independent computation framework; C++11
- Works for two graph types: primal and incidence graph
- Supports weighted model counting (WMC)

Experimental Work (Naive Implementation)

Instances

- 2585 instances from public benchmarks
- #SAT and WMC

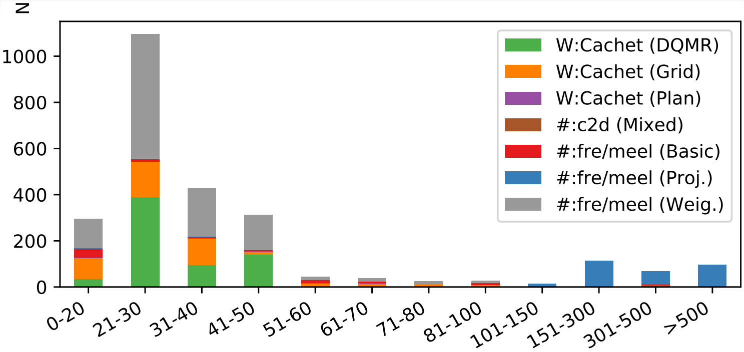
Limits

- Cannot expect to solve instances of high treewidth.

Hardware

- non-GPU solving: cluster of 9 nodes; each 2xE5-2650 CPUs(12cores) 2.2 GHz, 256 GB RAM; disabled HT, kernel 4.4
- GPU-solving: i3-3245 3.4 GHz; 16 GB RAM; GPU: Sapphire Pulse ITX Radeon RX 570 GPU; 1.24 GHz with 32 compute units, 2048 shader units, 4GB VRAM

Distribution of Primal Width



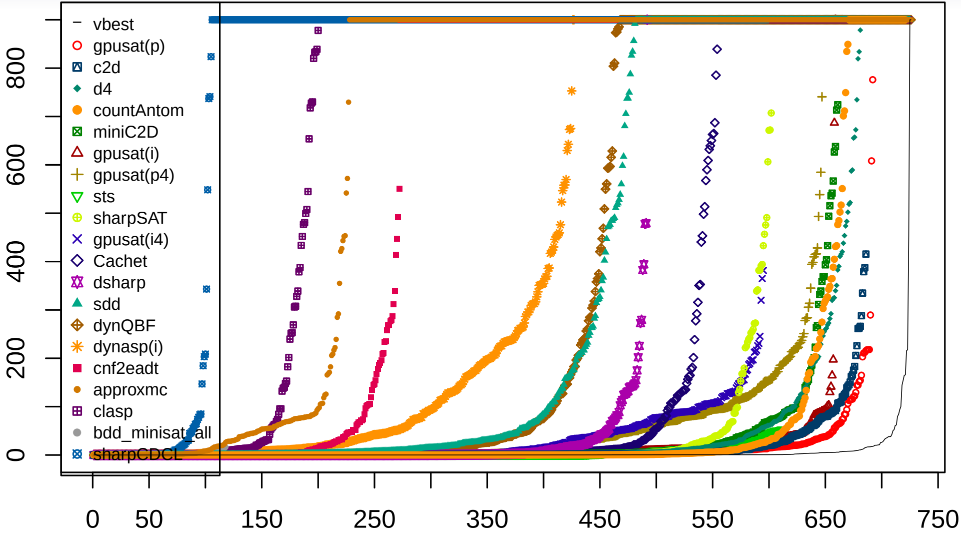
Decomposition Heuristic:

- Runtime well below a second (max. 2.5) 0–40
- Timeout (900s) on 41 instances

$\Rightarrow$ 54% primal treewidth below 30; 70% below 40

Parameterized Algorithms might work...

Solving (Width: 0–30): #SAT



Solving: #SAT

solver	0-20	21-30	31-40	41-50	51-60	>60	best	$\sum$	rank
c2d	164	519	**175**	116	20	118	120	1112	2
Cachet	133	421	91	109	8	58	13	820	7
d4	**169**	510	156	119	**23**	**162**	191	**1139**	1
gpusat(p)	**169**	**523**	79	104	0	0	88	875	6
miniC2D	167	491	137	103	8	67	2	973	4
sharpSAT	136	465	136	112	11	124	**483**	984	3
sts	162	448	101	**146**	10	45	252	912	5

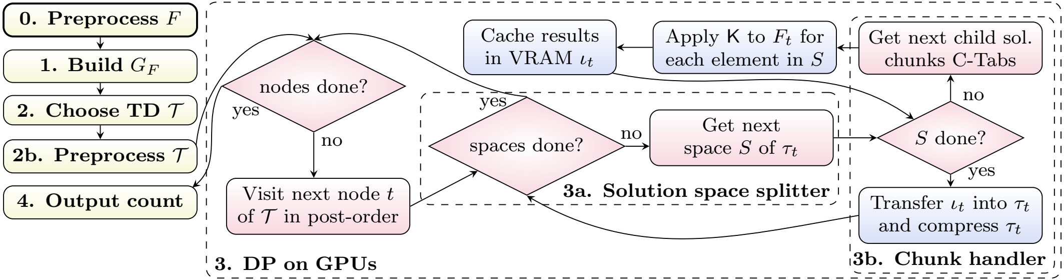
Table: Number of counting instances solved by solver and interval.

Empirical Work (first approach)

Observations

- Implementation is fairly naive
- Still: competitive up to width 30
- Requirement: obtain decompositions fast
- Width was surprisingly small (different for SAT)

New Architecture



0. Instance Preprocessing
2. Customized Tree Decompositions

3a. Solution Space Splitting

3b. Execute a small GPU-program in a GPU thread (kernel) for each element in S
Compress the data and store it in the VRAM (separate GPU-programs)
After all chunks are processed memory regions are merged

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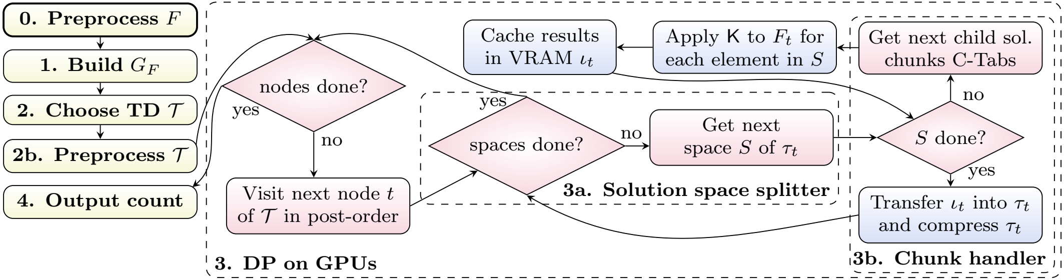

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   (#30; minimize max. card. of intersection of bags at node and its children)

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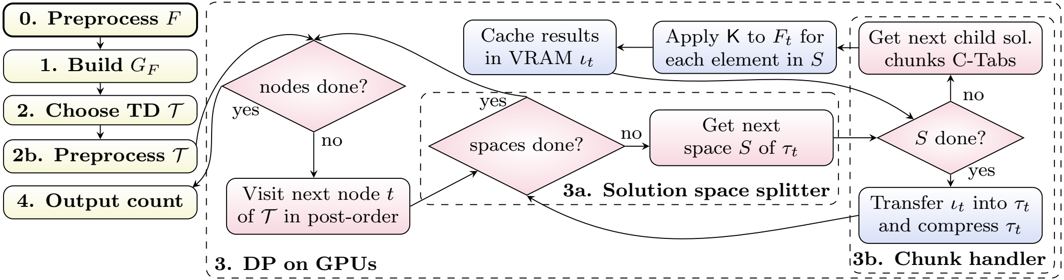

0. Instance Preprocessing
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3a. Solution Space Splitting
(Split larger solutions into smaller portions $\Rightarrow$ avoid OOM)
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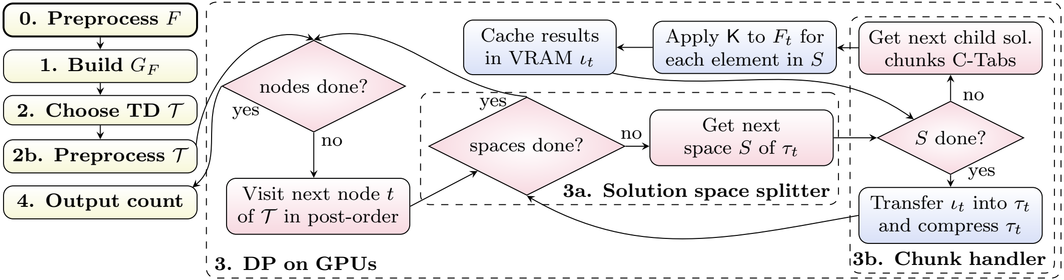

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Data Structures (Save Space)

Disclaimer for theorists: you need to get your hands dirty
(essentially: bit fiddling)

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1. Store compressed partial assignments
   (only where # $\neq 0$, simulate a BST in an array)
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1) Assignment Compression (BST in an Array)

- Continuous sequence 64-bit unsigned integers (cells)
- Cell: empty, index, and value (counter)
- index cells: lower 32 bits index to the next cell (variable $\rightarrow$ 0), upper bits (1)
- Handle Sync by keeping track of the current size
  (number of allocated cells; prevent to allocate cell again)

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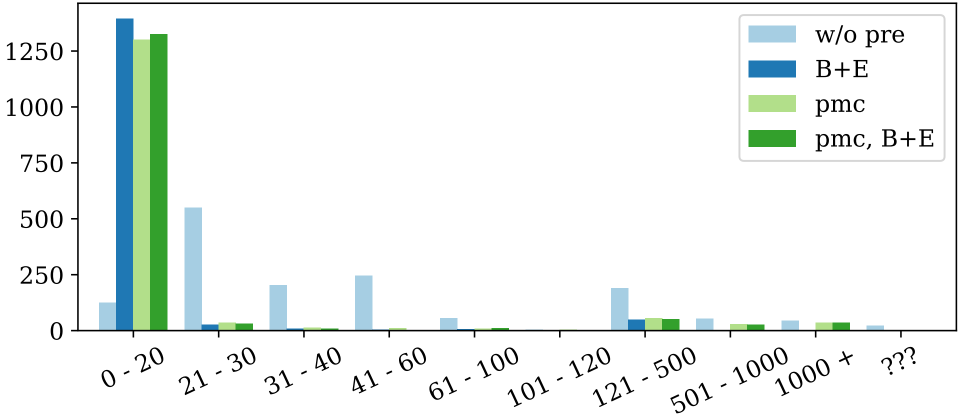
2) Data Type Precision

- Store floating log-counters
- Numbers stored in relation to exponent 2^{e} (largest exponent)
- Dynamically change exponent (keep highest possible precision)

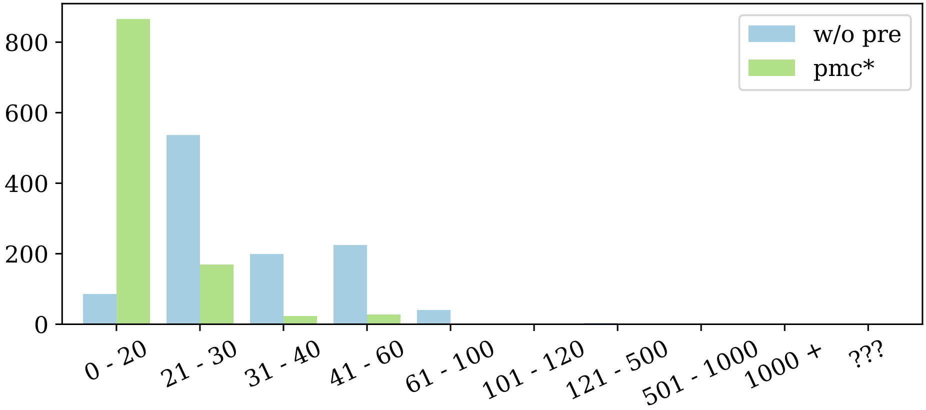
Where are we at with the new architecture?

#SAT: Width Comparison (w/o Preprocessing)



⇒ Produce decompositions of significantly smaller width

WMC: Width Comparison (w/o Preprocessing)



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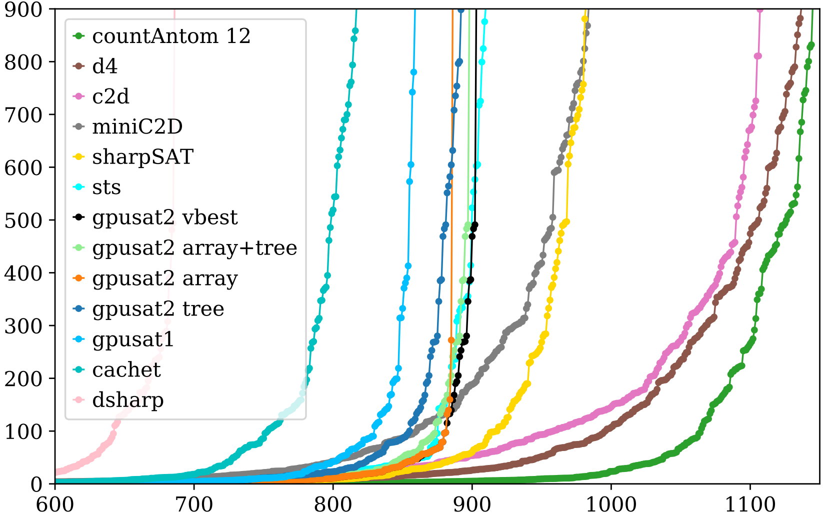
#SAT: Runtime Results (wo. Preprocessing)



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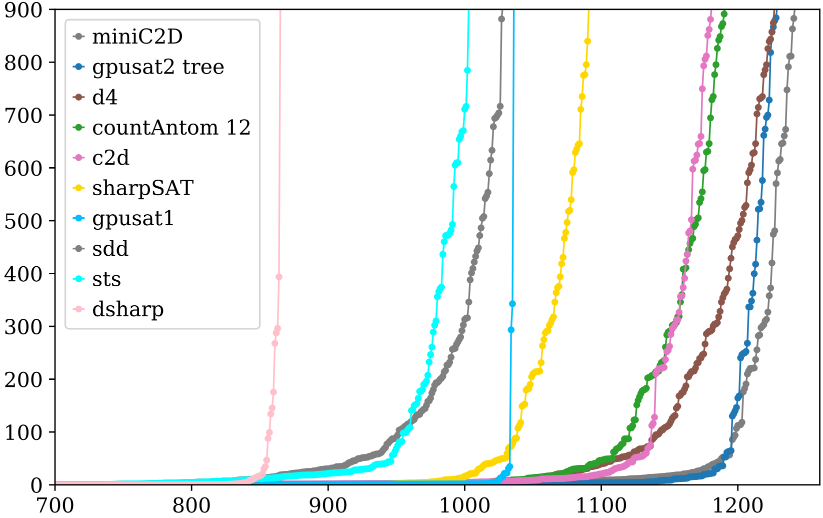
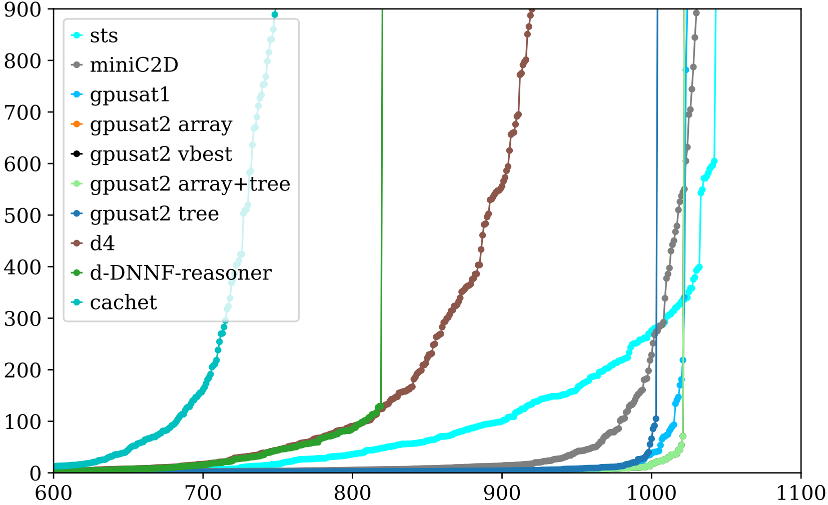


⇒ Techniques pay off after preprocessing

#WMC: Runtime Results (w. Preprocessing)



⇒: After preprocessing #SAT no3, WMC no2

Summary

Contributions

- Established Architecture for DP on the GPU
- Competitive Implementation for #SAT/WMC solving

Benchmark: Comparing apples and oranges

BUT: you compare parallel and sequential solvers.

1. We run on cheap consumer hardware (200 EUR).
2. Cannot measure speedup due to OpenCL limitations
   $\Rightarrow$ migrate to cuda

Summary contd.

Take Home Messages

1. Parameterized Algorithms can actually work
   (Preprocessing is key; some techniques pay only off with right preprocessing)
2. Does it work for SAT? $\Rightarrow$ we don't expect so.

Future Work

- Improve current setup by:
  Portfolio solving; Parallel Usage of GPUs; Alternative Frameworks
- Parameters (pswidth)

Sponsors: FWF Y698 & P26696; DFG HO 1294/11-1

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Thanks for listening!

GitHub:daajoe/{benchmark-tool,frasmt,trellis}

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