Weighted Model Counting on the GPU
by Exploiting Small Treewidth

Johannes K. Fichte\textsuperscript{1} \quad Markus Hecher\textsuperscript{2,3} \quad Stefan Woltran\textsuperscript{2} \quad Markus Zisser\textsuperscript{2}

\textsuperscript{1}TU Dresden, Germany
\textsuperscript{2}TU Wien, Austria
\textsuperscript{3}University of Potsdam, Germany

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Motivation

Model Counting (#SAT)

- Generalizes Boolean satisfiability problem (SAT)
- #SAT: output the number of satisfying assignments
- Various applications in AI and reasoning, e.g.,
  - Bayesian reasoning [Sang et al.’05]
  - Learning preference distributions [Choi et al.’15]
  - Infrastructure reliability [Meel et al.17]
- Computational complexity: #P-hard [Roth’96]
Problem of Interest

SAT-Problem (Boolean Satisfiability Problem)

Given: Propositional formula $F$.
Question: Is there a truth assignment $\tau$ to the variables in $F$ such that $F_\tau$ evaluates to 1 (satisfiable).

Input normal form

- Conjunctive normal form (CNF)
- Form: $F = (\ell_1 \lor \ell_2 \lor \ell_3) \land \ldots \land (\ldots)$ where $\ell_i$ either $x$ or $\neg x$

#SAT (Number SAT)

- Number of satisfying truth assignments to $F$. 
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**Example**

$$(a \lor \neg b \lor d) \land (\neg a \lor \neg c \lor \neg d) \land$$

$$(\neg a \lor \neg d) \land (b \lor c) \land (d \lor e) \land (\neg b \lor \neg e)$$

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⇒ Satisfiable

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Weighted Model Counting (WMC)

- Generalizes #SAT
- Given Boolean formula, e.g., \( F = (x \lor y) \land (\neg x \lor \neg y) \)

  Function that maps literals to reals between 0 and 1, e.g.,
  \( x \mapsto 0.4, \neg x \mapsto 0.6, y \mapsto 0.7, \neg y \mapsto 0.3 \)

- Weight of an assignment \( \alpha \) is the product over the weights of its literals, i.e.,
  \[
  w(\alpha) := \prod_{v \in \alpha^{-1}(1)} w(v) \cdot \prod_{v \in \alpha^{-1}(0)} w(\neg v),
  \]
  e.g., \( \alpha(x) = 1, \alpha(y) = 0 \Rightarrow w(\alpha) = 0.4 \cdot 0.3 = 0.16 \)

- Weighted model count (WMC) of formula is the sum of weights over all its satisfying assignments, e.g.,
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  w(F) = w(\alpha_1) + w(\alpha_2) = 0.12 + 0.42 = 0.54
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Theory:
SAT cannot be solved faster than $2^{o(n)}$ steps! (ETH)

Idea:
- Practical instances are usually highly structured
- Structure can be exploited by algorithms

⇒ Various Approaches in Solvers to exploit Structure
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Motivation: A somewhat different approach.

#SAT/WMC Solving

- There are already various solvers based on various techniques: approximate (Meel) / CDCL (Baccus/Thurley) / knowledge compilation based (Darwiche et al.)

Parameterized Algorithms

- Lots a theoretical work over last 20 years and various algorithms for #SAT

Research Question

Are (theoretical) algorithms from parameterized complexity even useful for implementations in #SAT/WMC solving?
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Parameterized Algorithmics

Topic of the Talk

Solve #SAT/WMC by means of an implementation of a parameterized algorithm that exploits small treewidth.
Tree Decompositions

**Treewidth**

- Most prominent graph invariant
- Small treewidth indicates tree-likeness and sparsity
- Can be used to solve #SAT/WMC by defining graph representations of the input formula
Treewidth

- Treewidth defined in terms of tree decompositions (TD)
- TD: arrangement of graph into a tree + bags s.t. ...
  - Treewidth: width of a TD of smallest width
Tree Decompositions

Treewidth ▶️ Definition & Example

- Treewidth defined in terms of tree decompositions (TD)
- TD: arrangement of graph into a tree + bags s.t. ...
- Treewidth: width of a TD of smallest width

Diagram:

- Graph: vertices labeled a, b, c, d, e
- Tree decomposition: nodes labeled b, d, b, d, b, a, c, d, e
- Width: the maximum size of the bags in the tree decomposition
Outline

1. Build graph $G$ of $F$
2. Create TD $T$ of $G$
3. Dynamic Programming
   - Store results in table $\tau_t$
   - Apply $A$ to $F_t$
   - done? no
     - Visit next node $t$ of $T$ in post-order
   - yes
4. Output count

Part:
A) Background & Basic Concepts
   - Treewidth, Graph Representation (1) + Dynamic Programming (3) [Samer & Szeider JDA’10]
B) Finding TDs (2)
C) Dynamic Programming (3) on the GPU
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“Find” tree decompositions of small width?

Works well even for relatively large instances.

Thanks to the Parameterized Algorithms and Computational Experiments Challenge (PACE) ’16/’17!!!
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How to “use” tree decompositions for #SAT/WMC?
Graph Representations

By Example

\[ v_a \lor v_b \lor v_c \]
\[ v_b \lor v_c \lor v_d \]
\[ v_d \lor v_e \]

Formula $F$

Primal graph
Graph Representations

By Example

\[ v_a \lor v_b \cdot v_a \lor v_c. \]
\[ v_b \lor v_c. v_b \lor v_d. \]
\[ v_d \lor v_e. \]

Formula $F$

Primal graph
Exploiting Tree Decompositions (TDs)

Dynamic Programming for SAT [Samer & Szeider’10]

1. Decompose graph
2. Algorithm for SAT
3. Combine solutions

Runtime: $\mathcal{O}(2^k \cdot \|F\|^2)$ where $F$ is the input formula and $k$ the width of TD
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A GPU-based #SAT/WMC-solver

OR how to go parallel?
How to parallelize DP?

1. Compute tables for multiple nodes in parallel
   - Does not allow for immediate massive parallelization due to dependencies to children

2. Distribute computation of rows among different computation units
   - Allows with right hindsight for massive parallelization

Why: computation of rows are independent
Dynamic Programming on the GPU

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Why: computation of rows are independent
Implementation (gpuSAT1)

Right hindsight?

- Data structures: a “pixel” represents #solutions (store data as array)
- Table merging: merge small bags (< 14)
- Table splitting: split large tables
- Weight: avoid double overflows by factor for #SAT (so it’s actually WMC with uniform weights).

Implementation

- OpenCL: vendor and hardware independent computation framework; C++11
- Works for two graph types: primal and incidence graph
- Supports weighted model counting (WMC)
Experimental Work (Naive Implementation)

Instances
- 2585 instances from public benchmarks
- #SAT and WMC

Limits
- Cannot expect to solve instances of high treewidth.

Hardware
- non-GPU solving: cluster of 9 nodes; each 2x E5-2650 CPUs (12 cores) 2.2 GHz, 256 GB RAM; disabled HT, kernel 4.4
- GPU-solving: i3-3245 3.4 GHz; 16 GB RAM; GPU: Sapphire Pulse ITX Radeon RX 570 GPU; 1.24 GHz with 32 compute units, 2048 shader units, 4 GB VRAM
Distribution of Primal Width

Decomposition Heuristic:

- Runtime well below a second (max. 2.5) 0–40
- Timeout (900s) on 41 instances
  ⇒ 54% primal treewidth below 30; 70% below 40

Parameterized Algorithms might work...
Solving: #SAT

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<th>41-50</th>
<th>51-60</th>
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</tr>
</tbody>
</table>

Table: Number of counting instances solved by solver and interval.
Empirical Work (first approach)

Observations

- Implementation is fairly naive
- Still: competitive up to width 30
- Requirement: obtain decompositions fast
- Width was surprisingly small (different for SAT)
0. Instance Preprocessing

2. Customized Tree Decompositions

3a. Solution Space Splitting

3b. Execute a small GPU-program in a GPU thread (kernel) for each element in $S$

Compress the data and store it in the VRAM (separate GPU-programs)

After all chunks are processed memory regions are merged
0. Instance Preprocessing
2. Customized Tree Decompositions
   (#30; minimize max. card. of intersection of bags at node and its children)
3a. Solution Space Splitting
3b. Execute a small GPU-program in a GPU thread (kernel) for each element in $S$; Compress the data and store it in the VRAM (separate GPU-programs); After all chunks are processed memory regions are merged
0. Instance Preprocessing
2. Customized Tree Decompositions
3a. Solution Space Splitting
   (Split larger solutions into smaller portions ⇒ avoid OOM)
3b. Execute a small GPU-program in a GPU thread (kernel) for each element in $S$
   Compress the data and store it in the VRAM (separate GPU-programs)
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New Architecture

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2. Customized Tree Decompositions
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   Compress the data and store it in the VRAM (separate GPU-programs)
   After all chunks are processed memory regions are merged
Data Structures (Save Space)

Disclaimer for theorists: you need to get your hands dirty (essentially: bit fiddling)
1. Store compressed partial assignments
   (only where \( \neq 0 \), simulate a BST in an array)
2. Use logarithmic counters
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1) Assignment Compression (BST in an Array)
   - Continuous sequence 64-bit unsigned integers (cells)
   - Cell: empty, index, and value (counter)
   - index cells: lower 32 bits index to the next cell (variable $\rightarrow 0$), upper bits (1)
   - Handle Sync by keeping track of the current size
     (number of allocated cells; prevent to allocate cell again)
Data Structures (Save Space)

1. Store compressed partial assignments
   (only where $# \neq 0$, simulate a BST in an array)
2. Use logarithmic counters

2) Data Type Precision
   - Store floating log-counters
   - Numbers stored in relation to exponent $2^e$ (largest exponent)
   - Dynamically change exponent (keep highest possible precision)
Where are we at with the new architecture?
#SAT: Width Comparison (w/o Preprocessing)

⇒ Produce decompositions of significantly smaller width
WMC: Width Comparison (w/o Preprocessing)

Produce decompositions of significantly smaller width
#SAT: Runtime Results (wo. Preprocessing)
#SAT: Runtime Results (w. Preprocessing)

Techniques pay off after preprocessing
# WMC: Runtime Results (w. Preprocessing)

≫: After preprocessing #SAT no3, WMC no2
Summary

Contributions

- Established Architecture for DP on the GPU
- Competitive Implementation for #SAT/WMC solving

Benchmark: Comparing apples and oranges

BUT: you compare parallel and sequential solvers.

1. We run on cheap consumer hardware (200 EUR).
2. Cannot measure speedup due to OpenCL limitations
   ⇒ migrate to cuda
Take Home Messages

1. Parameterized Algorithms can actually work  
   (Preprocessing is key; some techniques pay only off with right preprocessing)
2. Does it work for SAT? ⇒ we don’t expect so.

Future Work

- Improve current setup by: 
  Portfolio solving; Parallel Usage of GPUs; Alternative Frameworks
- Parameters (pswidth)

Sponsors: FWF Y698 & P26696; DFG HO 1294/11-1
Take Home Messages

1. Parameterized Algorithms can actually work
   (Preprocessing is key; some techniques pay only off with right preprocessing)
2. Does it work for SAT? ⇒ we don’t expect so.

Future Work

- Improve current setup by:
  - Portfolio solving; Parallel Usage of GPUs; Alternative Frameworks
- Parameters (pswidth)

Thanks for listening!

Advertisement:
PACE-2019 (vertex cover and hypertree decompositions)
GitHub: daajoe/{benchmark-tool,frasmt,trellis}

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