Finding Hamiltonian Cycle in Graphs of Bounded Treewidth: Experimental Evaluation[1]

Marcin Pilipczuk, Michał Ziobro

March 12, 2019

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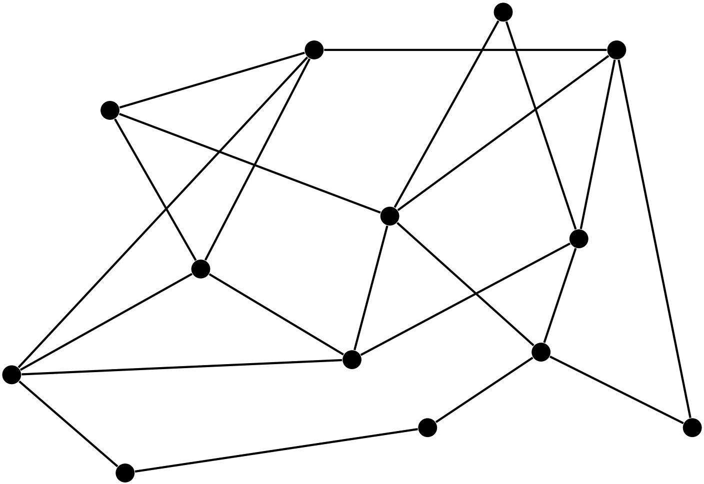
Separator



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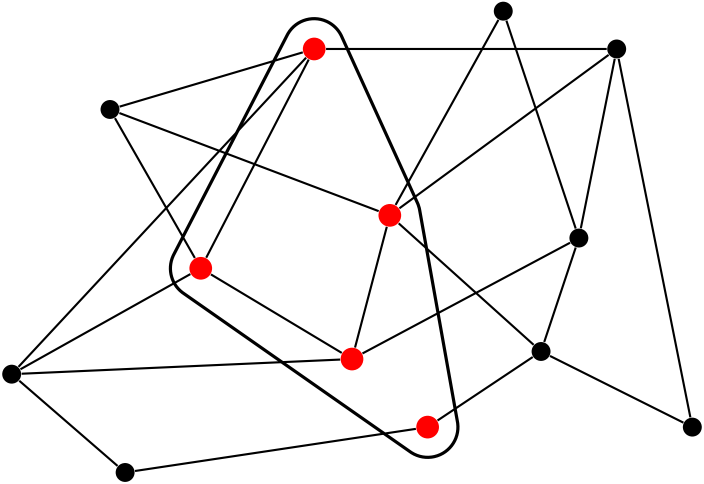

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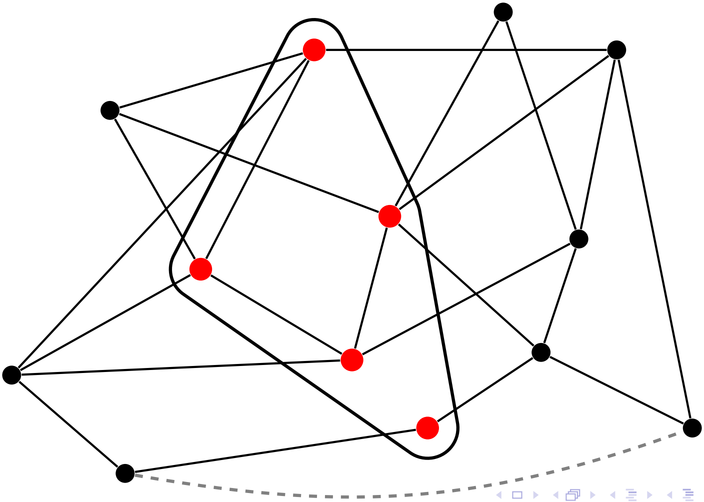

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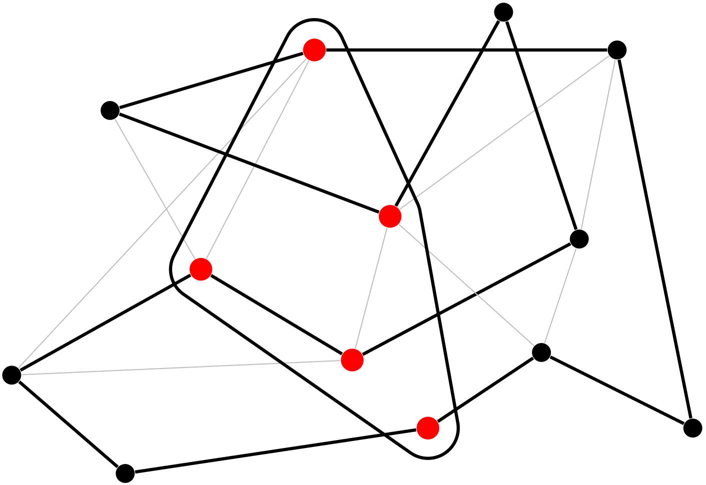

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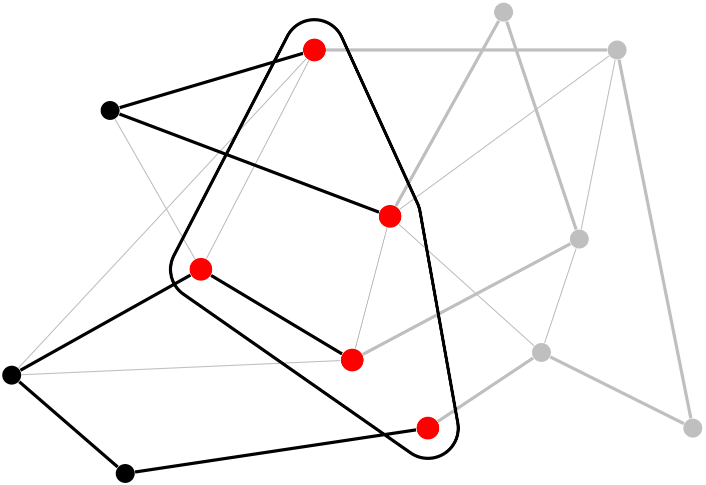

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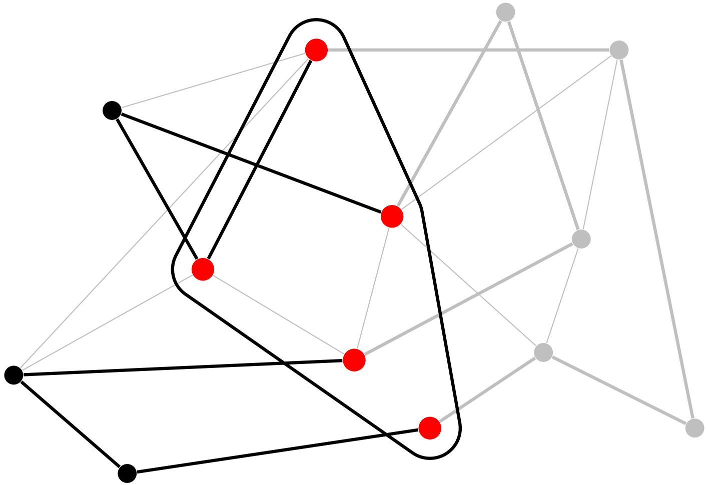
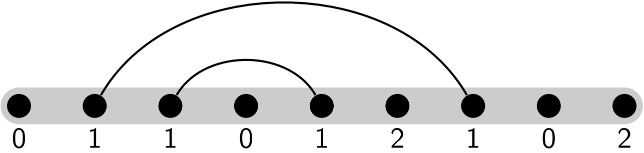


Fingerprints



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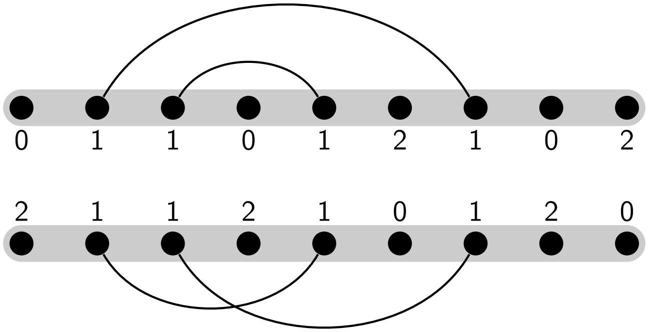

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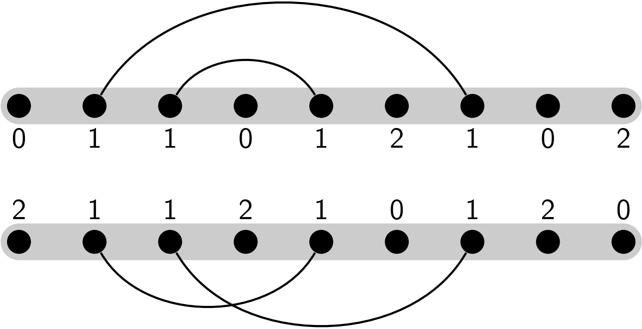

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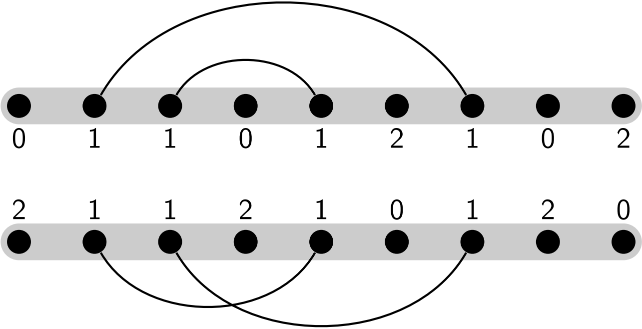

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k vertices of degree 1 $\Rightarrow$ $k!!$ possible fingerprints

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tree decomposition — set of separators covering whole graph
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FPT dynamic algorithm with running time $2^{O(t \ln t)} O(n^c)$, where $t =$ treewidth

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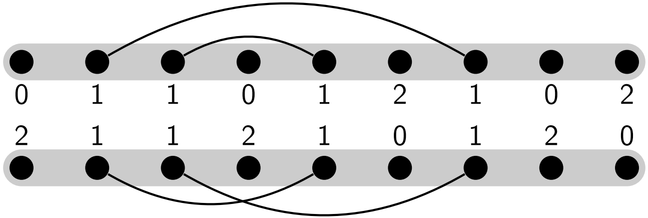

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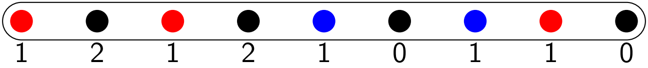

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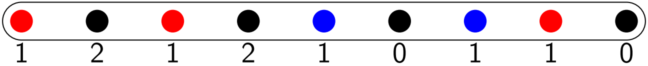

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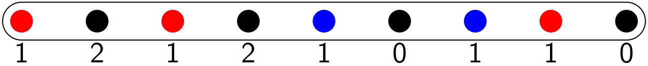

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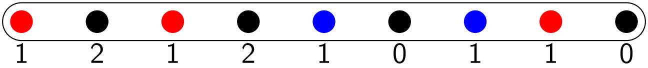

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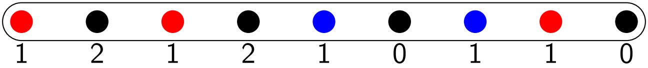

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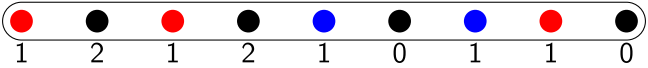

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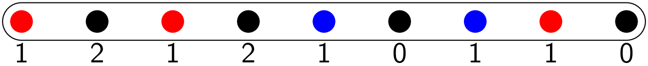

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- naive join nodes: 9^t,
- transform at join nodes: 4^t, but problems with $GF(2^{64})$.

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- A - instances with small treewidth from FHCP Challenge
- B - randomly sampled subset of A (for adjusting hyperparameters)
- C - instances with treewidth between 17 and 29 from FHCP Challenge
- D - subset of C which were solved by at least one of our algorithms (for adjusting hyperparameters)
- E - our few random instances

Small treewidth results

- **1st rank-based**: 18.59% times slower than naive,
- **2nd rank-based**: 10.97% faster,
- **cut-and-count**: solved 499 from 623 instances (TL: 600s)

test	$\|V\|$	tw	naive	rank-based 1	rank-based 2	c&c
0556	3274	9	20.655	27.794	28.024	128.231
0728	4170	9	30.861	38.578	38.823	279.871
0947	6598	9	128.733	143.144	142.427	467.181
0584	3411	9	105.371	114.240	73.291	-
0746	4286	9	631.261	619.601	381.351	-
0778	4561	8	17.468	16.974	13.069	-
0950	6620	9	196.572	206.482	124.641	-

Large treewidth results

test	$\lvert V \rvert$	tw	naive	rank-based 1	rank-based 2	c&c
0074	462	28	38.737	109.655	110.040	-
0253	1578	29	93.343	167.458	167.440	-
0268	1644	25	36.449	70.157	69.111	-
0272	1662	25	554.271	1260.329	1230.722	-
0298	1806	23	10.035	18.611	18.492	-
0172	1002	25	1.156	1.298	.554	-
0199	1200	25	13.513	15.419	3.369	-
E0002	600	18	204.197	-	28.882	-
E0003	700	20	-	-	711.778	-
E0007	360	15	1575.475	-	328.191	-

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- For sparse graphs with a large treewidth a cost of dividing fingerprints into families is often greater than gain from reducing number of them.
- Cut-and-count approach is impractical.
- **Conjecture**: reducing only classes with 4 vertices of degree one may be the best.