Networks of Multi-Server Queues with Parallel Processing

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Introduction

Datacenters

- Specialized servers
- Massively parallel processing
- Highly variable job requirements

Examples

- Computer clusters
- Content Delivery Networks

Objective: Resource allocation with predictable performance

Background on Order Independent queues

Multi-server queues with parallel processing

Scheduling algorithm

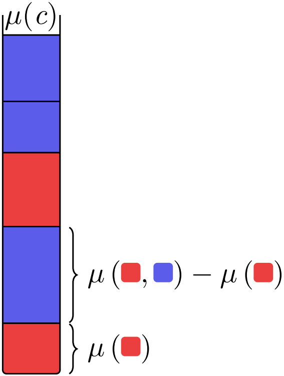
Queue state

- Finite set of job classes $I = \{$🟦, 🟥$\}$
  - Poisson external arrivals per class
  - Exponential job sizes with unit mean
- Queue state = sequence of job classes, ordered by arrival

$c = ($🟥, 🟦, 🟥, 🟦, 🟦$)$

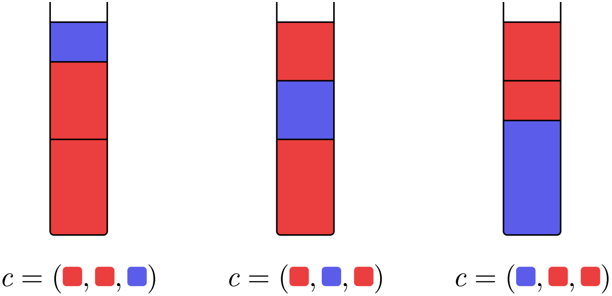
↑ head ↑ tail

- Total service rate in state $c = \mu(c)$



Service rate

- **Normalized:** $\mu(\emptyset) = 0$ and $\mu(c) > 0$ for any state $c \neq \emptyset$
- **Non-decreasing:** $\mu(c) \leq \mu(c, i)$ for any state c and class i
- **Order-independent:**



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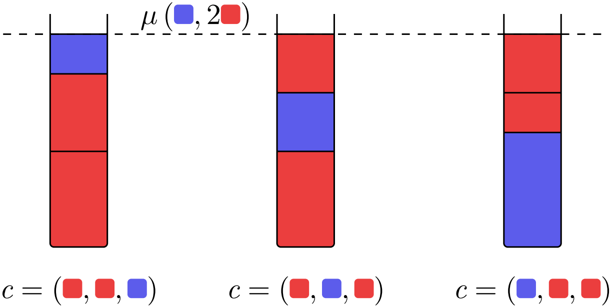

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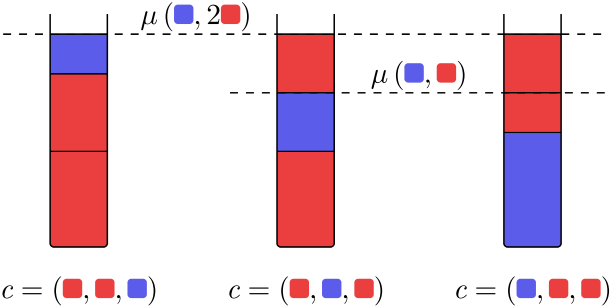

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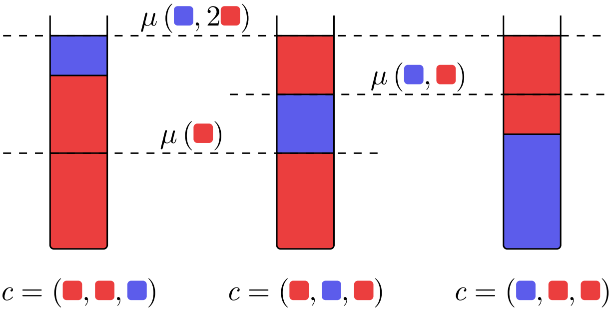


Order Independent queues (Berezner and Krzesinski, 1996)

- A stationary measure of the state $c = (c_1, \dots, c_n)$ is

$$\pi(c) = \pi(\emptyset) \prod_{k=1}^{n} \frac{\lambda_{c_k}}{\mu(c_1, \dots, c_k)}, \quad \forall c \in I^*$$

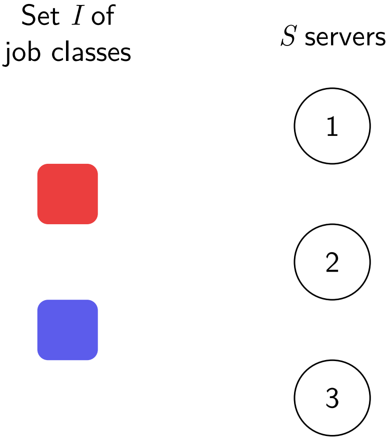
- The queue is **quasi-reversible**
  - The current state of the queue is independent of previous departures and future arrivals
  - Arrivals and departures form independent Poisson processes

Background on Order Independent queues

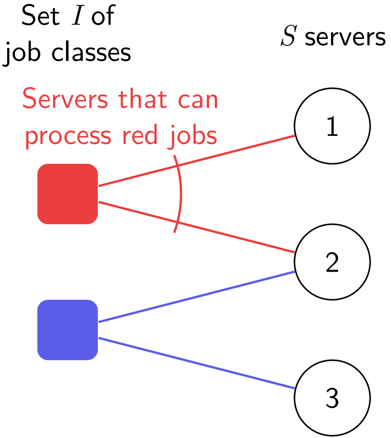
Multi-server queues with parallel processing

Scheduling algorithm

Server assignment



Server assignment



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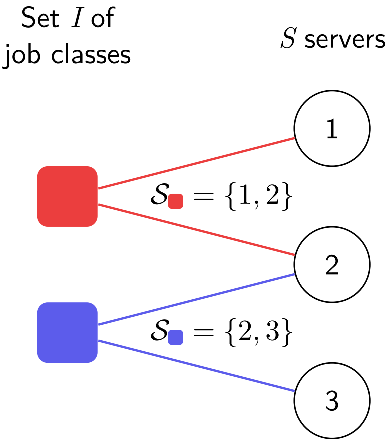
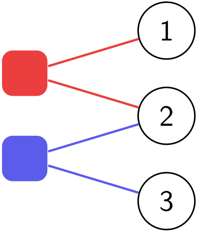


Service discipline



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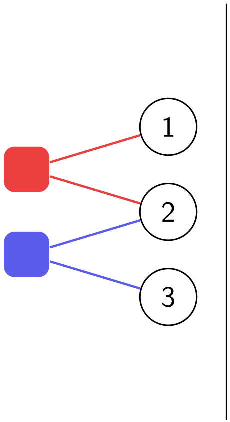

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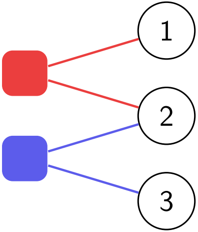
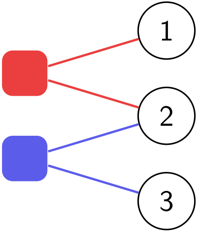


- Parallel processing

Service discipline



- Parallel processing
- First-come first-served per server

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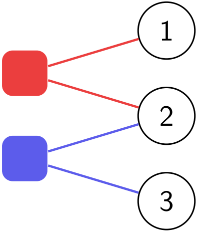

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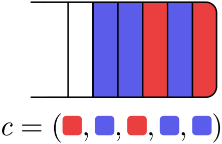
State of the queue



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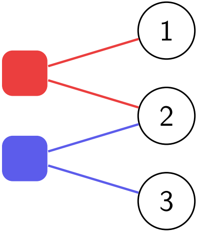

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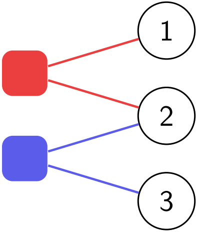

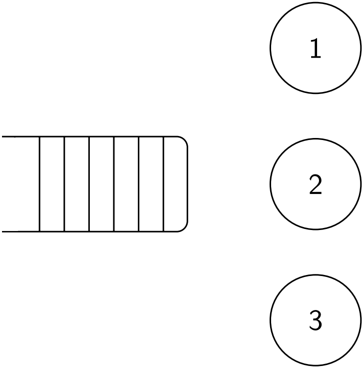

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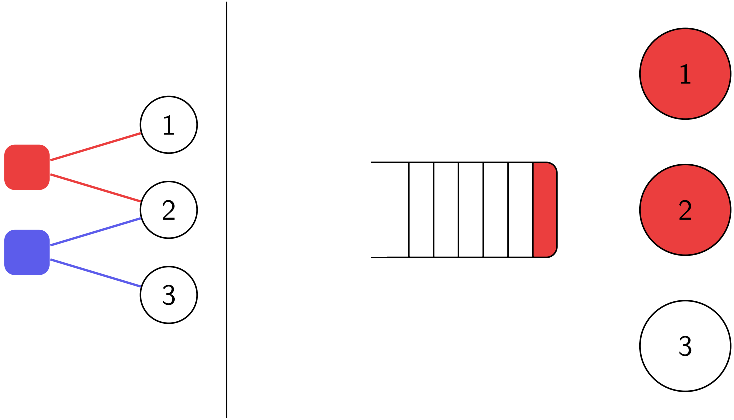

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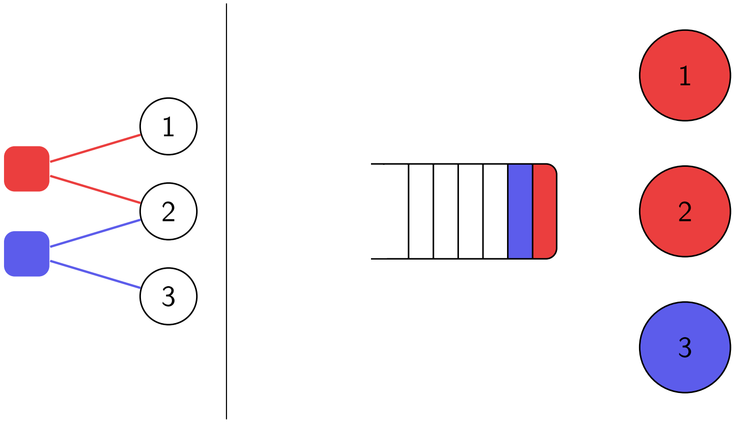

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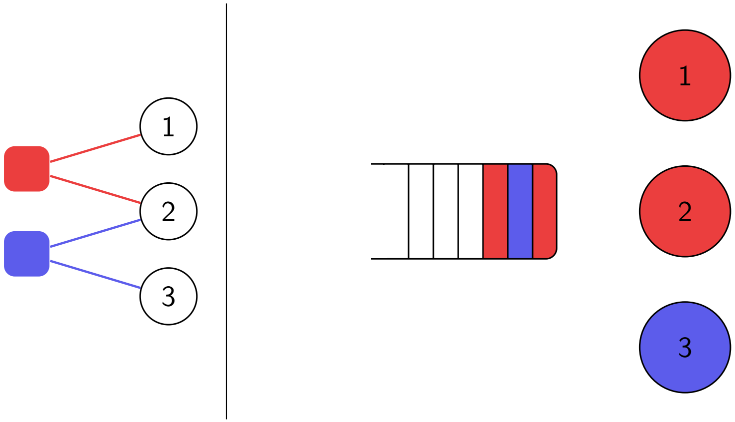

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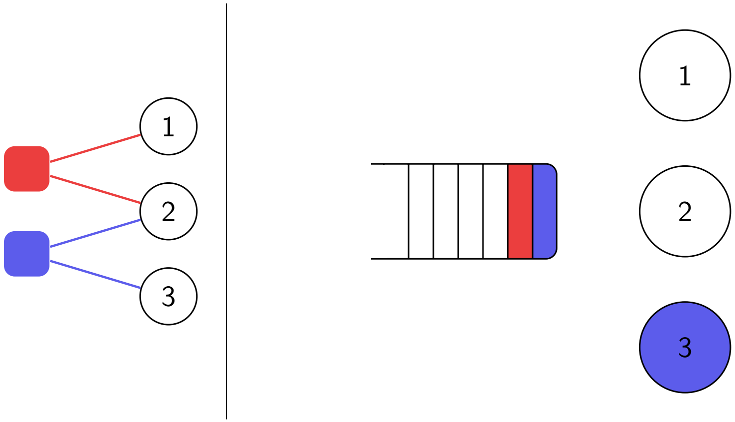

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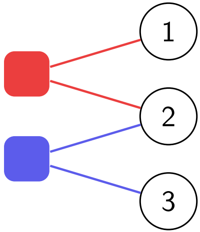
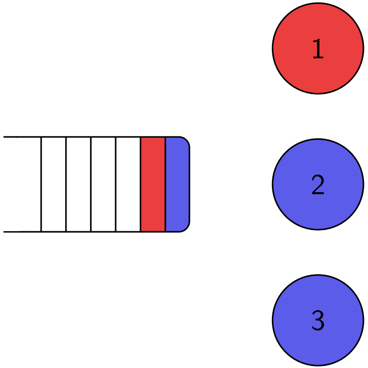
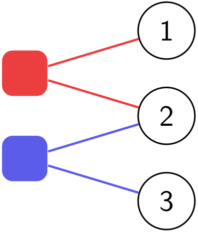




Service rate



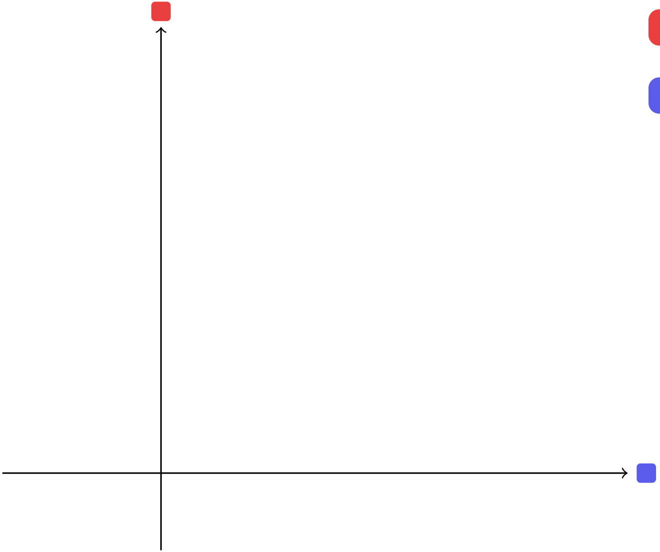
- Reinterpretation of (Gardner et al., 2015)
- μ only depends on the set of active classes

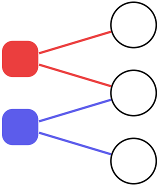
$$\mu(A) = \sum_{s \in \bigcup_{i \in A} \mathcal{S}_i} \text{capacity of server } s$$

- μ is submodular: if $A \subset B$ and $i \notin B$,

$$\mu(A \cup \{i\}) - \mu(A) \geq \mu(B \cup \{i\}) - \mu(B)$$

Service rate





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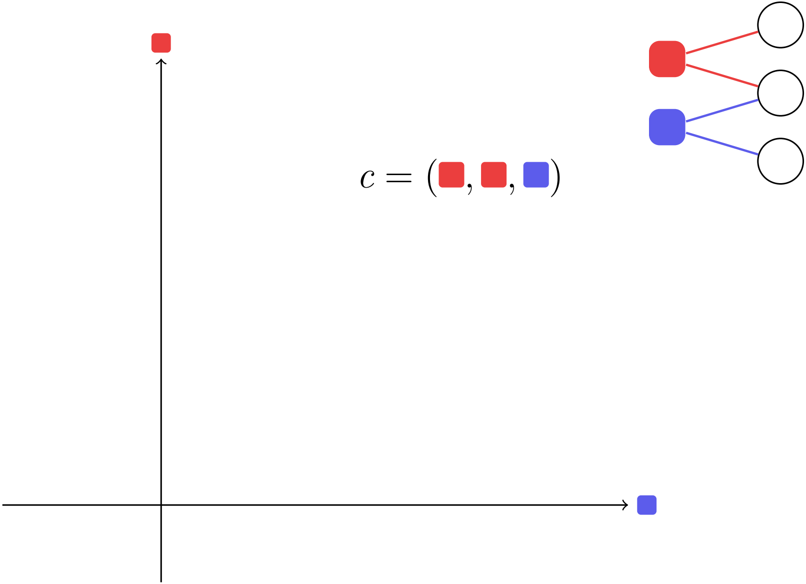

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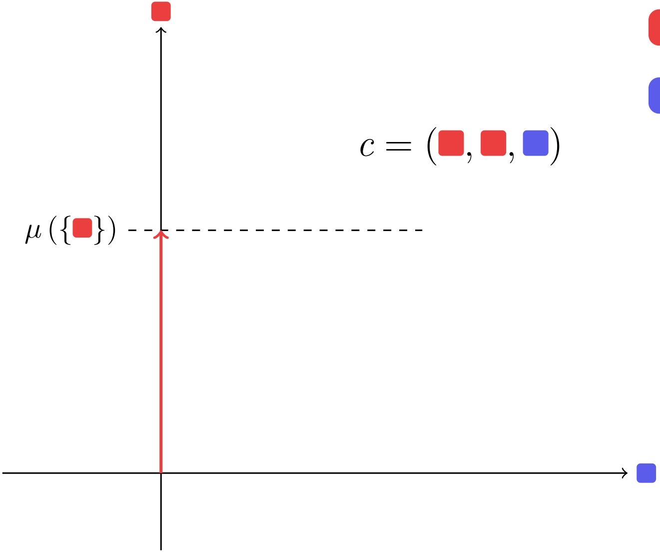
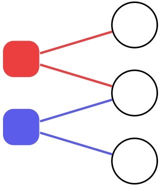


$$c = (■, ■, ■)$$



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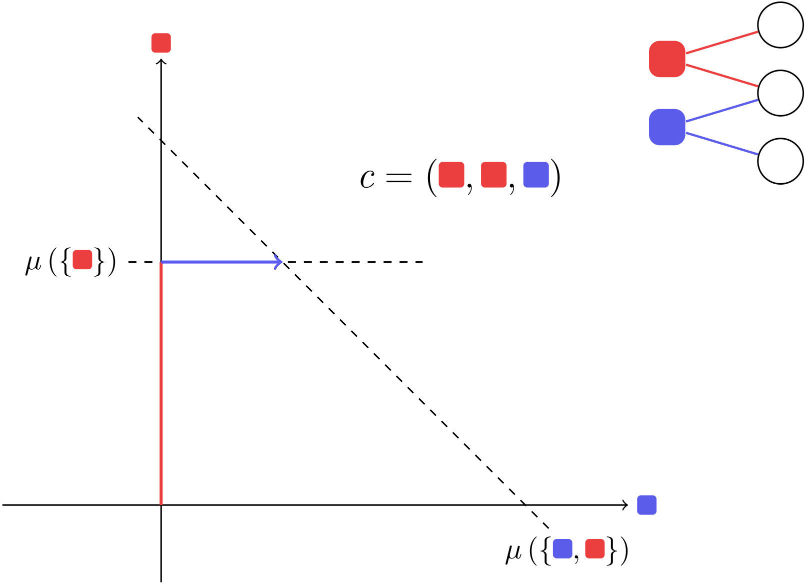

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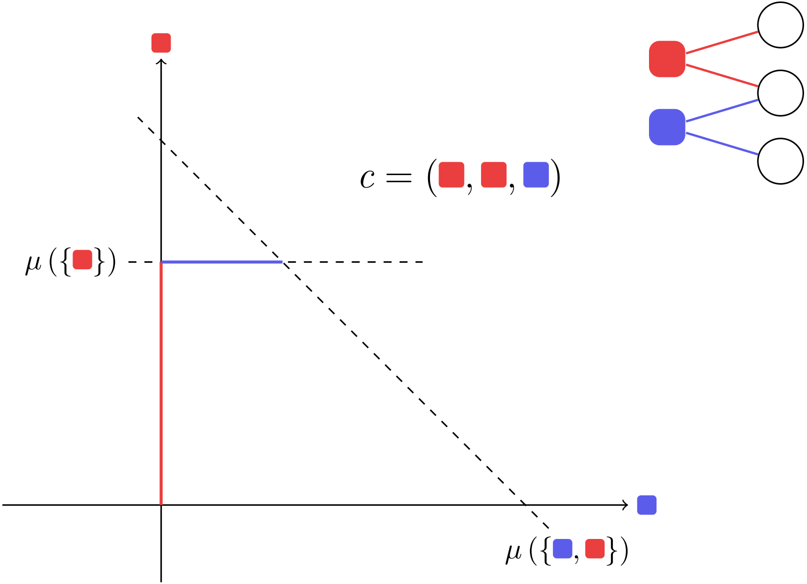

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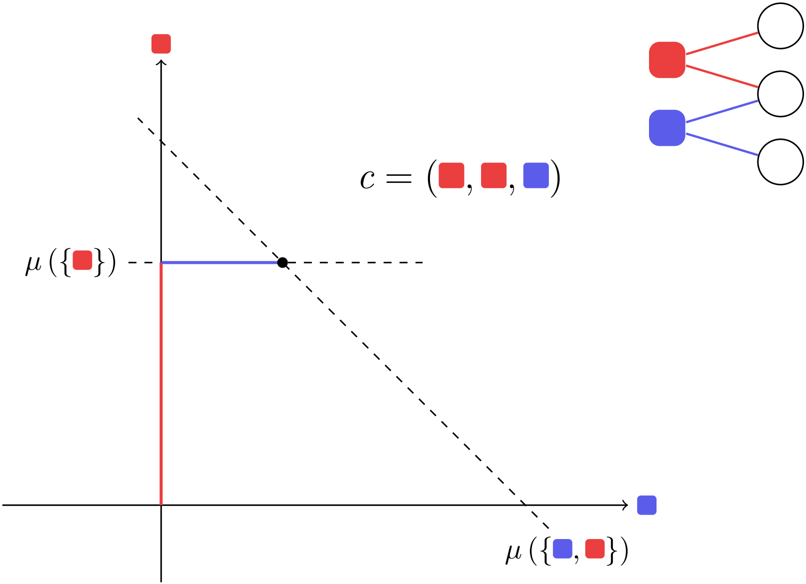

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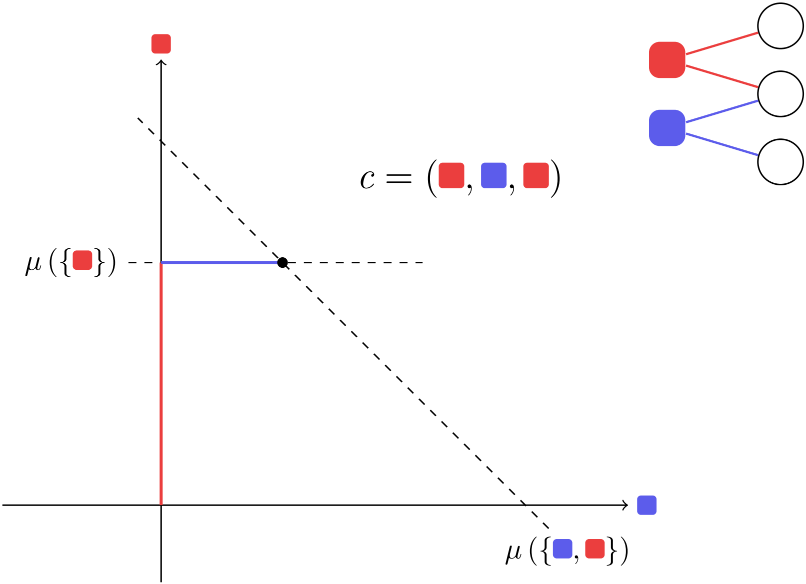

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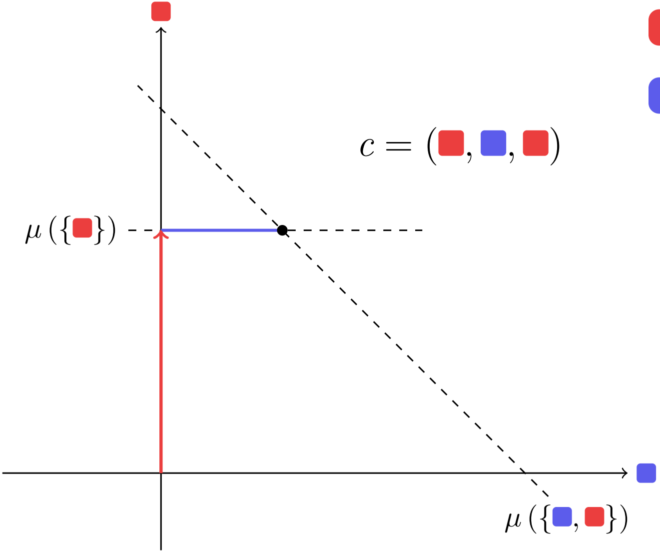

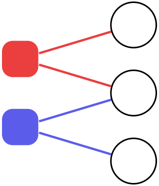

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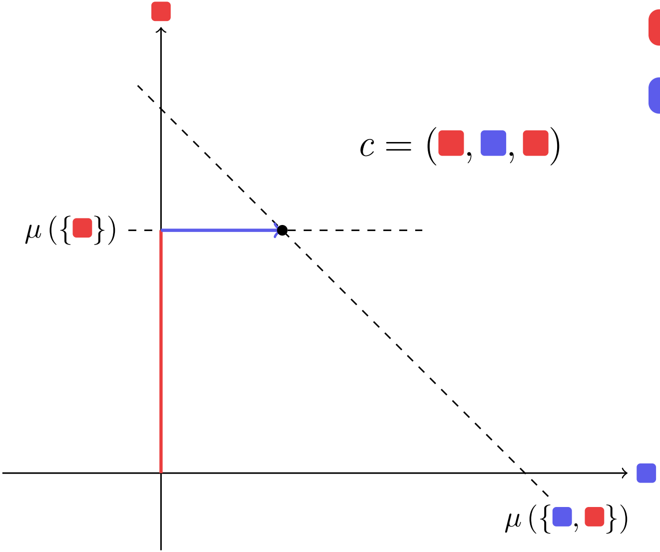

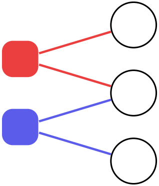

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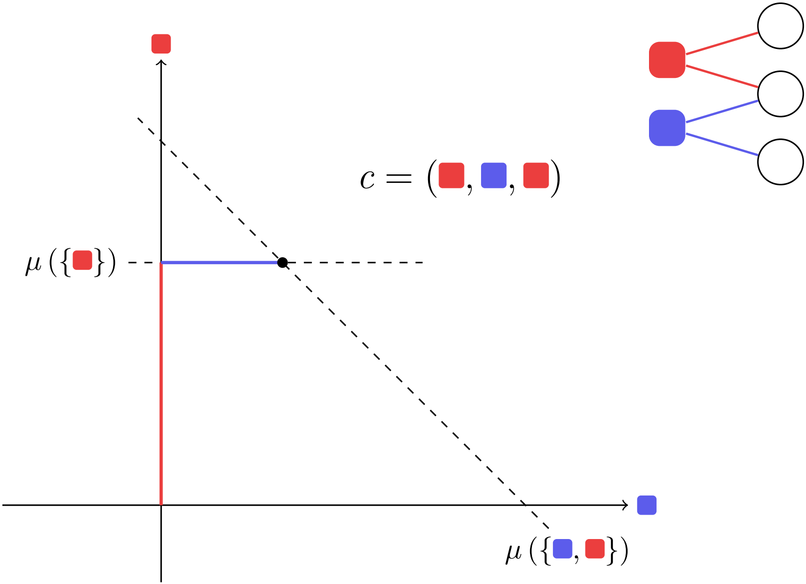

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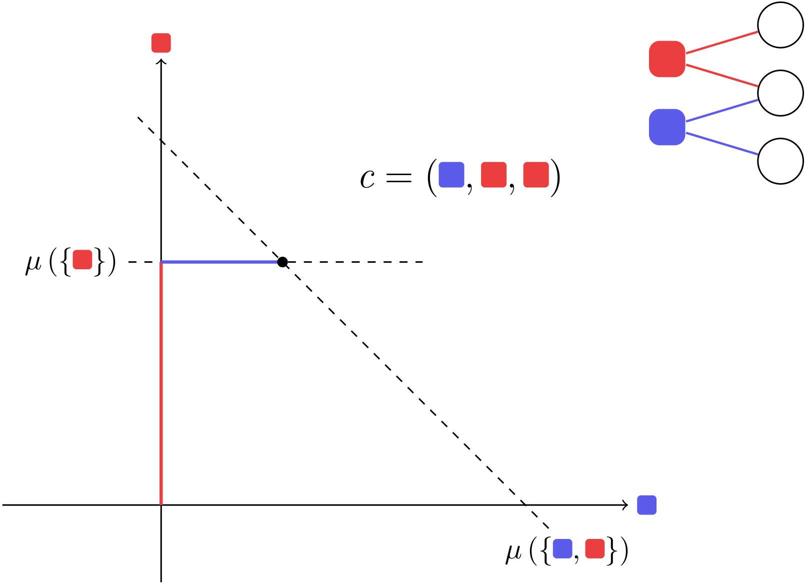

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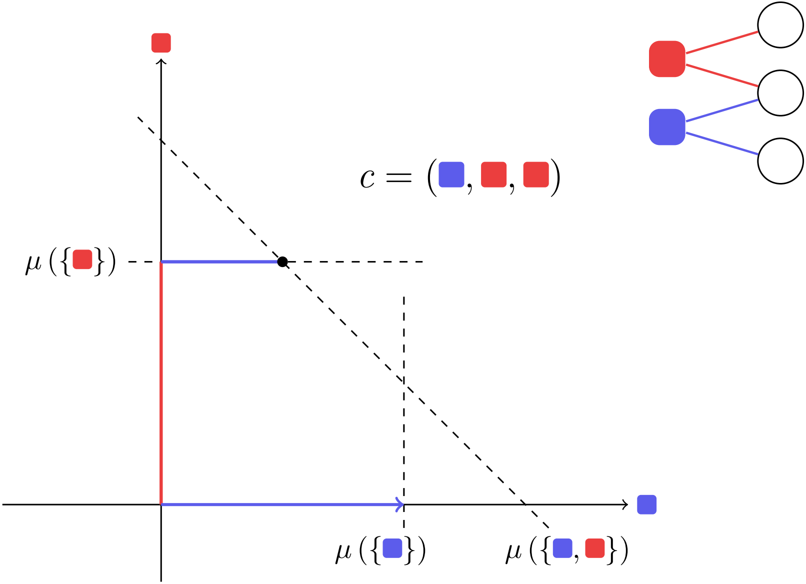

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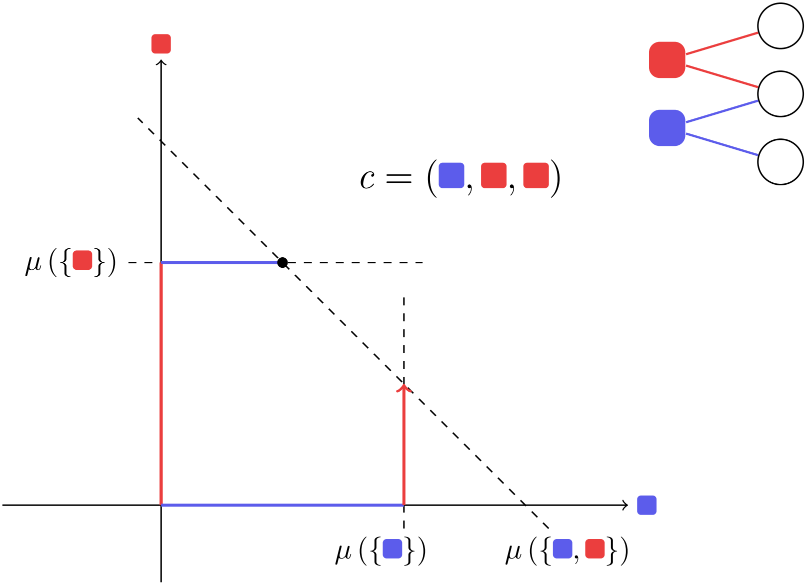

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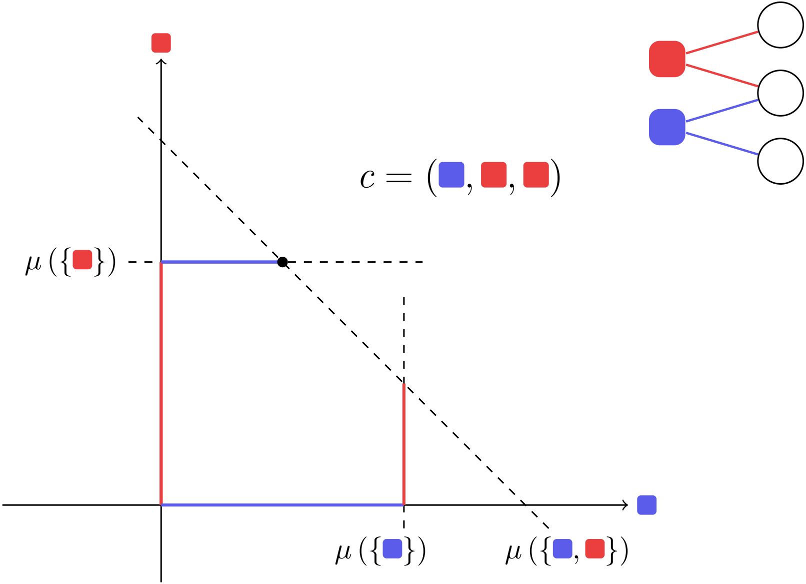

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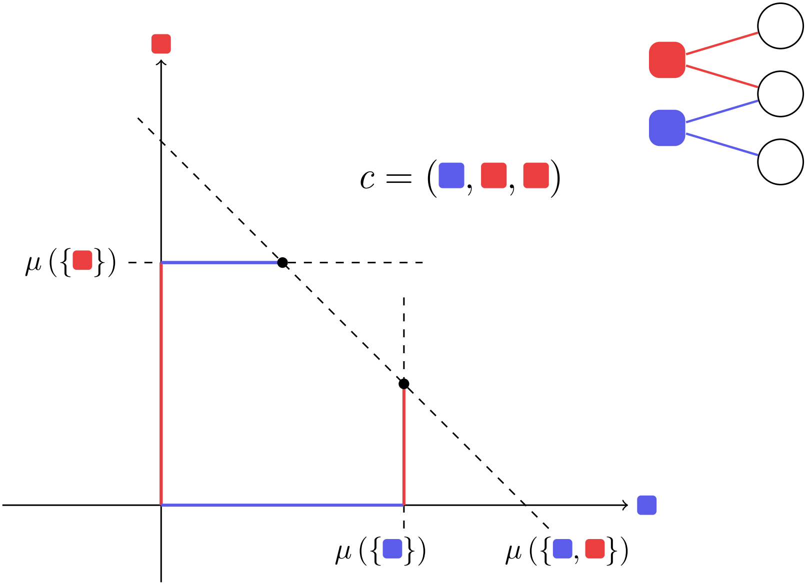

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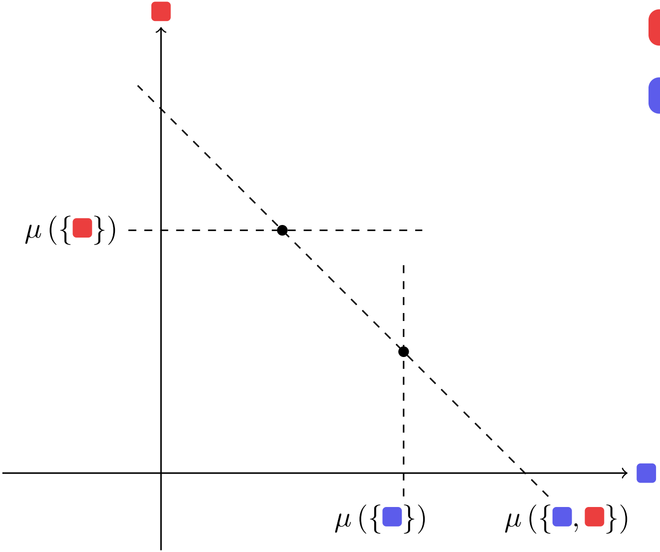

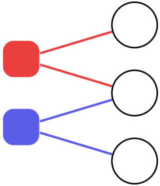

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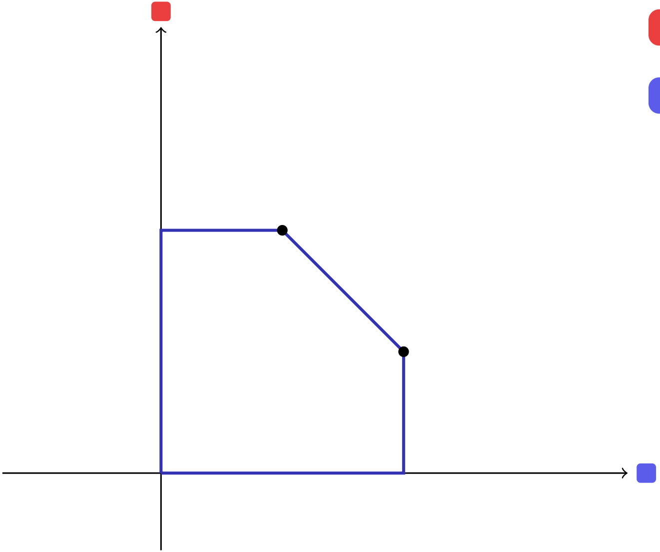

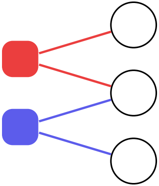

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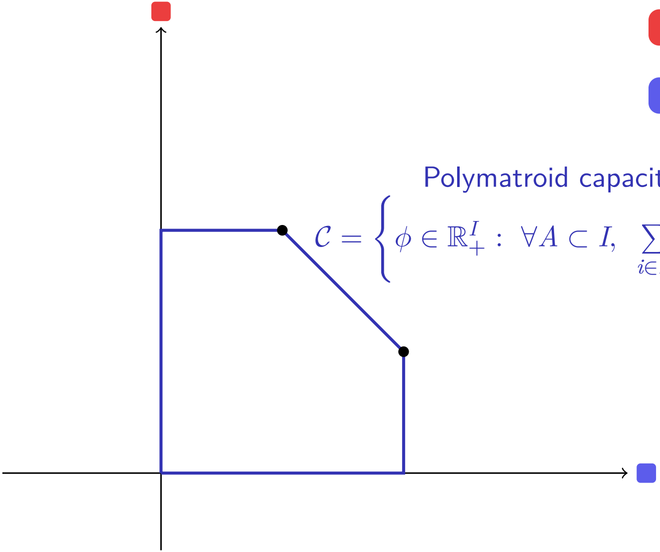
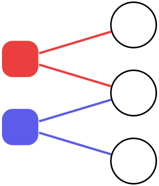




Polymatroid capacity set

$$\mathcal{C} = \left\{ \phi \in \mathbb{R}_+^I : \forall A \subset I, \sum_{i \in A} \phi_i \leq \mu(A) \right\}$$

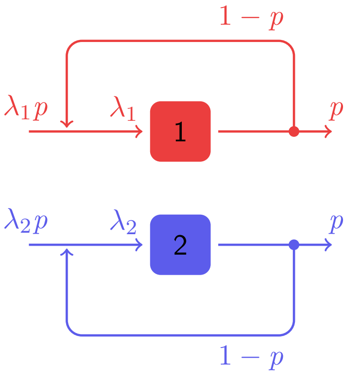
Stationary measure (Berezner and Krzesinski, 1996)

- A stationary measure of the queue state is

$$\pi(c) = \pi(\emptyset) \prod_{k=1}^{n} \frac{\lambda_{c_k}}{\mu(A(c_1, \ldots, c_k))}, \quad \forall c \in I^*$$

- The queue is **quasi-reversible**
  - The current state of the queue is independent of previous departures and future arrivals
  - Arrivals and departures form independent Poisson processes

Internal routing



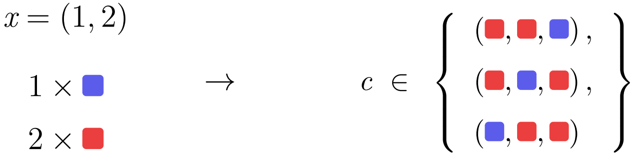
By quasi-reversiblity, the stationary measure of the queue state is

$$\pi(c) = \pi(\emptyset) \prod_{k=1}^{n} \frac{\lambda_{c_k}}{\mu(A(c_1, \ldots, c_k))},$$

independently of $p \in [0, 1]$.

State aggregation

Aggregate state $x = (x_i : i \in I) \in \mathbb{N}^I$



Stationary measure $\pi(x) = \sum_{c : |c| = x} \pi(c)$

Stationary distribution (Berezner and Krzesinski, 1996)

The stationary measure of the aggregate state x satisfies

$$\pi(x) = \pi(0)\Phi(x) \prod_{i \in I} {\lambda_i}^{x_i}, \quad \forall x \in \mathbb{N}^I,$$

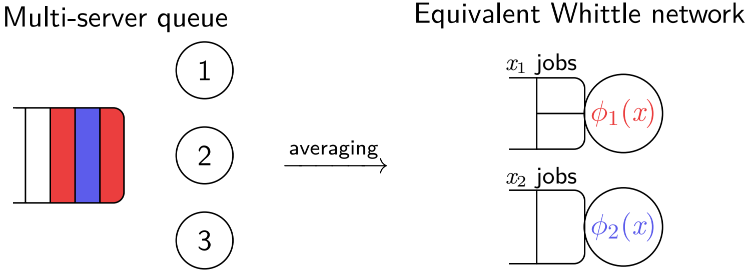
where Φ is defined by the recursion $\Phi(0) = 1$ and, for each $x \neq 0$,

$$\Phi(x) = \frac{1}{\mu(A(x))} \sum_{i \in A(x)} \Phi(x - e_i).$$

Equivalent Whittle network

π is the stationary measure of the state of a **Whittle network** of $|I|$ queues with arrival rates $\lambda_i, i \in I$, and service rates

$$\phi_i(x) = \frac{\Phi(x - e_i)}{\Phi(x)}, \quad \forall x \in \mathbb{N}^I, \quad \forall i \in A(x).$$



Equivalent Whittle network

Per-class service rates:

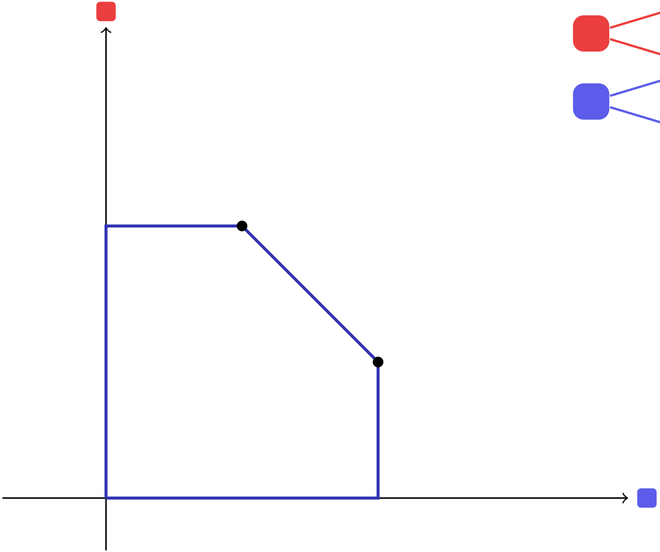
- $\mu_i(c)$ in state c of the multi-server queue
- $\phi_i(x)$ in state x of the Whittle network

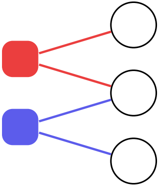
Theorem 1

The service rates in the equivalent Whittle network are the average per-class service rates in the multi-server queue:

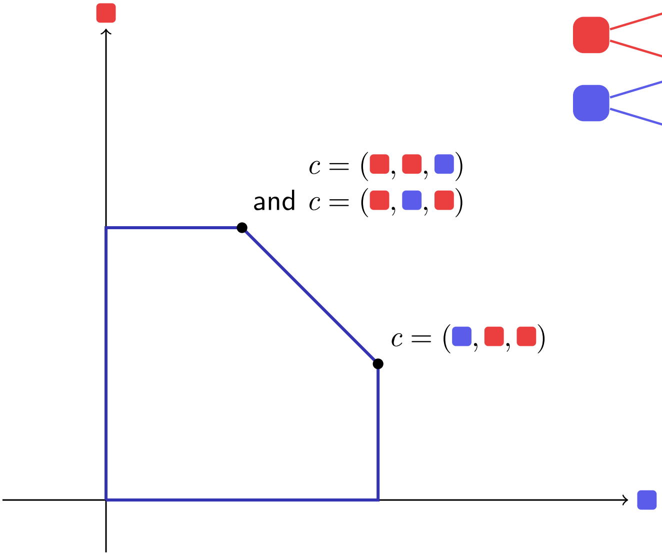
$$\phi_i(x) = \sum_{c:|c|=x} \frac{\pi(c)}{\pi(x)} \mu_i(c), \quad \forall x \in \mathbb{N}^I, \quad \forall i \in A(x).$$

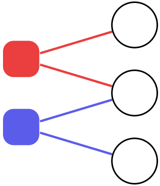
Average service rates





Average service rates





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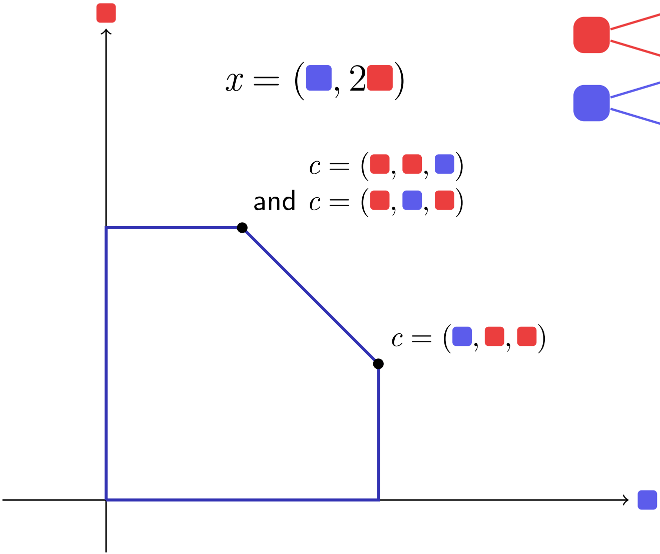

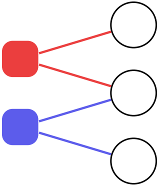

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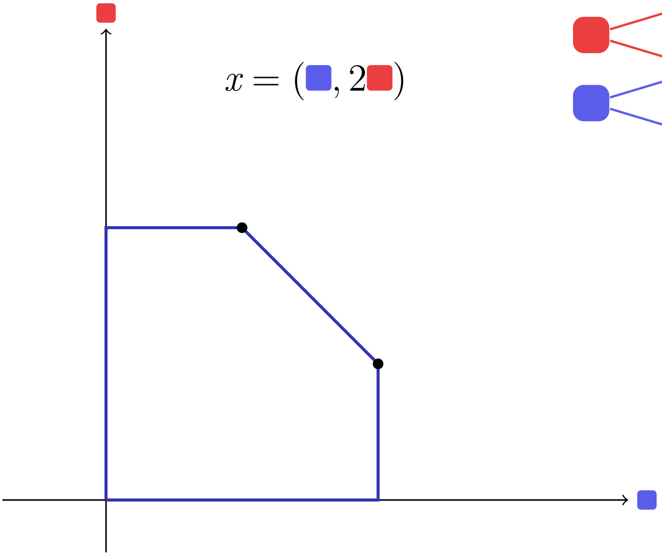
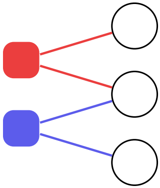
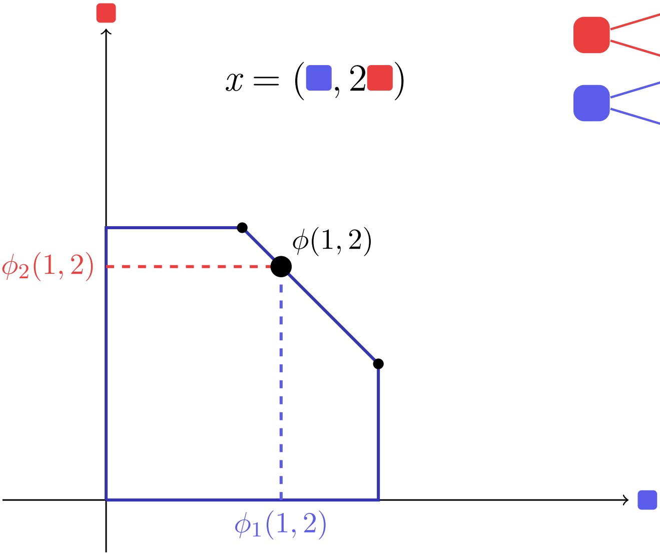
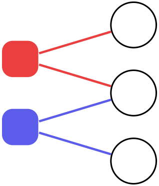




Average service rates





Balanced fairness

The average service rates are

- Balanced: $\dfrac{\phi_i(x-e_j)}{\phi_i(x)} = \dfrac{\phi_j(x-e_i)}{\phi_j(x)}, \quad \forall x \in \mathbb{N}^I, \quad \forall i,j \in A(x),$
- Efficient: $\displaystyle\sum_{i \in I} \phi_i(x) = \mu(A(x)), \quad \forall x \in \mathbb{N}^I.$

Balanced fairness in the capacity set

$$\mathcal{C} = \left\{ \phi \in \mathbb{R}_+^I : \quad \forall A \subset I, \quad \sum_{i \in A} \phi_i \le \mu(A) \right\}$$

Balanced fairness

The most efficient **insensitive** resource allocation

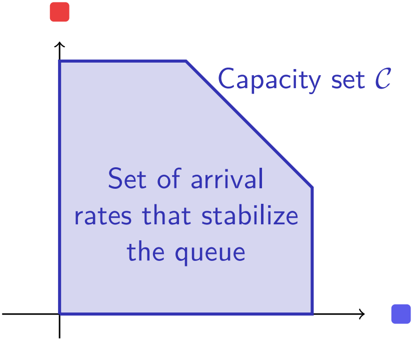
- Introduced for dimensioning data networks (Bonald and Proutière, 2003)
- Good approximation of proportional fairness
- Recently applied to Content Delivery Networks (Shah and de Veciana, 2015, 2016)

Stability condition

Theorem 2

The multi-server queue is stable if and only if

$$\forall A \subset I, \quad \sum_{i \in A} \lambda_i < \mu(A)$$



Aggregation

- Queue state c: $\pi(c) = \frac{\lambda_{c_n} \pi(c_1, \ldots, c_{n-1})}{\mu(c)}, \quad \forall c \neq \emptyset$
- Aggregate state x: $\pi(x) = \frac{\sum_{i \in A(x)} \lambda_i \pi(x - e_i)}{\mu(x)}, \quad \forall x \neq 0$
- Set of active classes A
  (Bonald et al., 2003; Shah and de Veciana, 2015, 2016)

$$\pi(A) = \frac{\sum_{i \in A} \lambda_i \pi(A \setminus \{i\})}{\mu(A) - \sum_{i \in A} \lambda_i}, \quad \forall A \neq \emptyset$$

$\rightarrow$ Closed-form expressions for the performance metrics

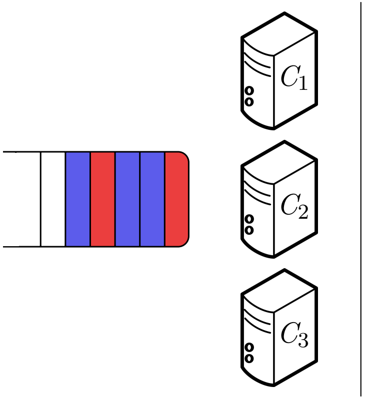
- Proportion of time the queue is idle
- Mean number of jobs of each class
- ...

Background on Order Independent queues

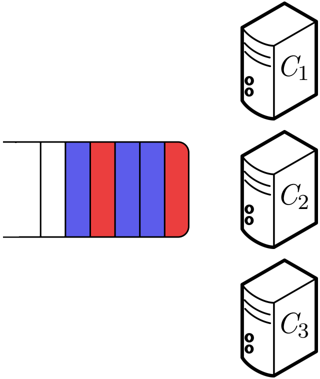
Multi-server queues with parallel processing

Scheduling algorithm

Computer cluster

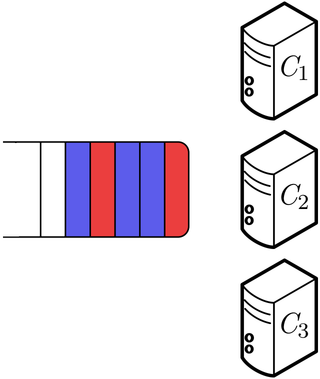


Computer cluster



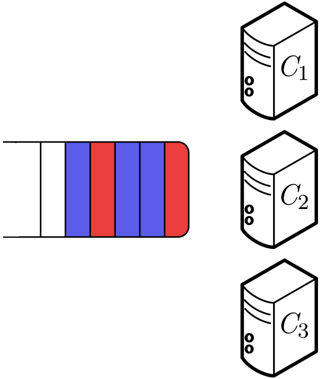
- Server s
  Fixed capacity C_s in flops

Computer cluster



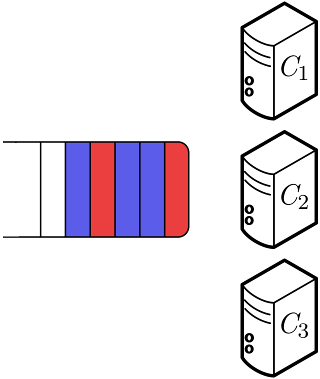
- Server s
  Fixed capacity C_s in flops
- An arriving job is assigned a set of computers *independently of the state of the cluster*

Computer cluster



- Server s
  Fixed capacity C_s in flops
- An arriving job is assigned a set of computers *independently of the state of the cluster*
- Service requirements
  General distribution with mean σ in floating-point operations

Computer cluster

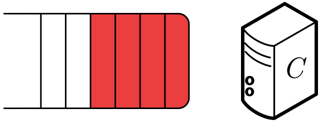


- Server s
  Fixed capacity C_s in flops
- An arriving job is assigned a set of computers *independently of the state of the cluster*
- Service requirements
  General distribution with mean σ in floating-point operations

Objective: Enforce balanced fairness

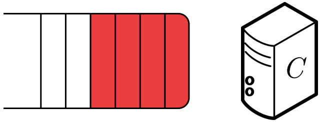
Single computer

Single-server mono-class cluster, service requirements with mean σ



Single computer

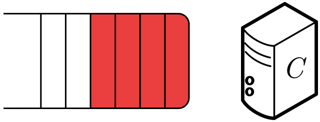
Single-server mono-class cluster, service requirements with mean σ



- FCFS at job scale
  Very sensitive

Single computer

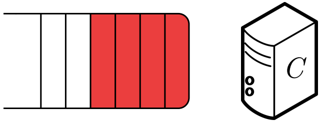
Single-server mono-class cluster, service requirements with mean σ



- FCFS at job scale
  Very sensitive
- Service interruption after θ floating point operations on average

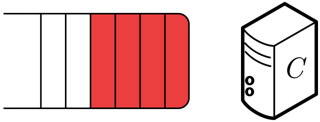
Single computer

Single-server mono-class cluster, service requirements with mean σ



Single computer

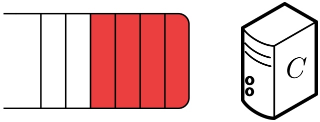
Single-server mono-class cluster, service requirements with mean σ



- Initialize a timer $\sim \mathcal{E}\left(\frac{C}{\theta}\right)$

Single computer

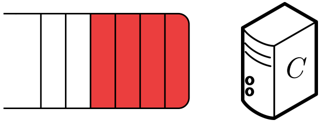
Single-server mono-class cluster, service requirements with mean σ



- Initialize a timer $\sim \mathcal{E}\left(\frac{C}{\theta}\right)$
- Upon service completion, the job leaves the cluster

Single computer

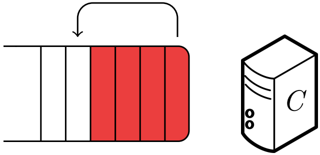
Single-server mono-class cluster, service requirements with mean σ



- Initialize a timer $\sim \mathcal{E}\left(\frac{C}{\theta}\right)$
- Upon service completion, the job leaves the cluster
- When the timer expires, the service is interrupted

Single computer

Single-server mono-class cluster, service requirements with mean σ



- Initialize a timer $\sim \mathcal{E}\left(\frac{C}{\theta}\right)$
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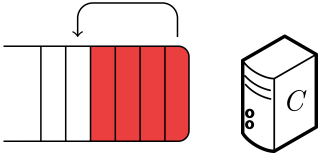
Single computer

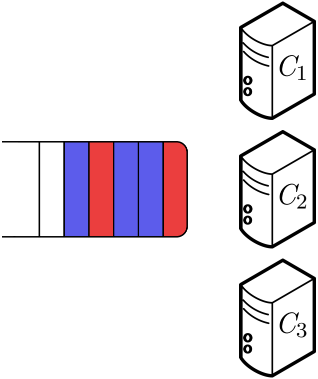
Single-server mono-class cluster, service requirements with mean σ



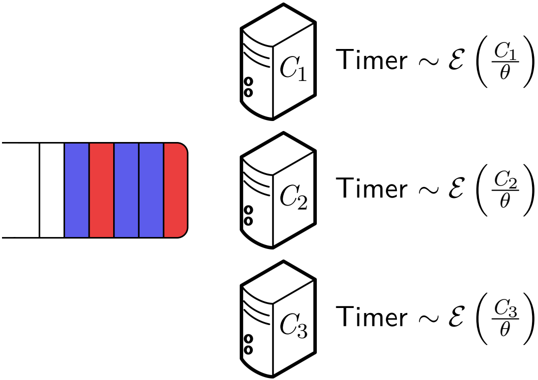
- Initialize a timer $\sim \mathcal{E}\left(\frac{C}{\theta}\right)$
- Upon service completion, the job leaves the cluster
- When the timer expires, the service is interrupted

Parameter $m =$ mean number of interruptions per job
When $m \to \infty$, single-server queue under PS service discipline

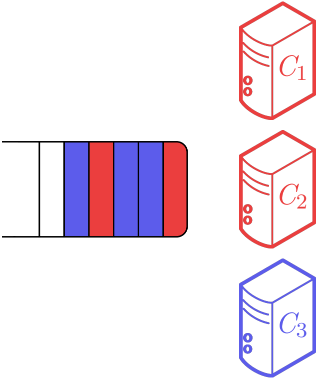
Computer cluster



Computer cluster



Computer cluster

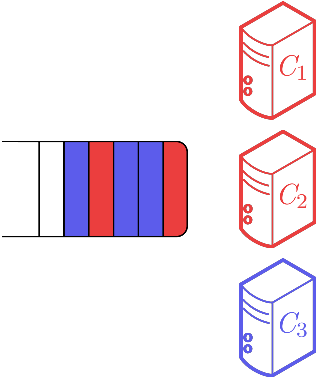


Timer $\sim \mathcal{E}\left(\frac{C_1}{\theta}\right)$

Timer $\sim \mathcal{E}\left(\frac{C_2}{\theta}\right)$

Timer $\sim \mathcal{E}\left(\frac{C_3}{\theta}\right)$

Computer cluster



Timer $\sim \mathcal{E}\left(\frac{C_1}{\theta}\right)$

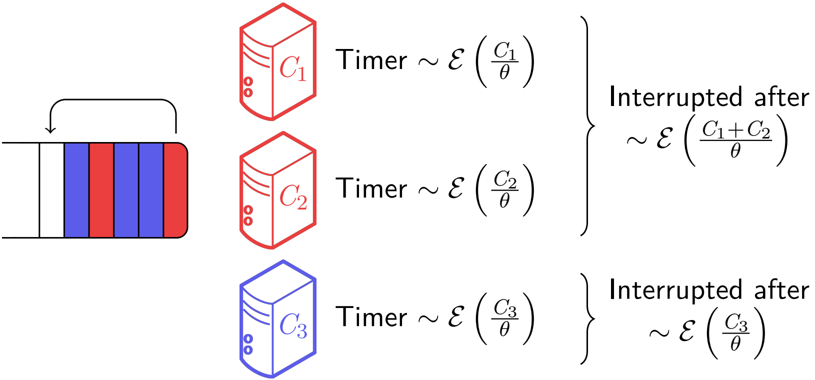
Timer $\sim \mathcal{E}\left(\frac{C_2}{\theta}\right)$

Interrupted after $\sim \mathcal{E}\left(\frac{C_1+C_2}{\theta}\right)$

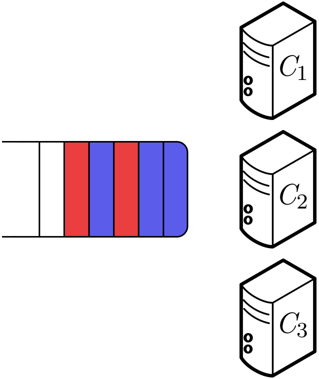
Timer $\sim \mathcal{E}\left(\frac{C_3}{\theta}\right)$

Interrupted after $\sim \mathcal{E}\left(\frac{C_3}{\theta}\right)$

Computer cluster



Computer cluster

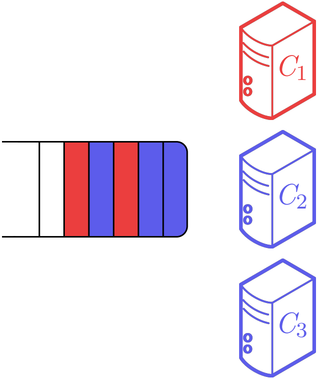


Timer $\sim \mathcal{E}\left(\frac{C_1}{\theta}\right)$

Timer $\sim \mathcal{E}\left(\frac{C_2}{\theta}\right)$

Timer $\sim \mathcal{E}\left(\frac{C_3}{\theta}\right)$

Computer cluster

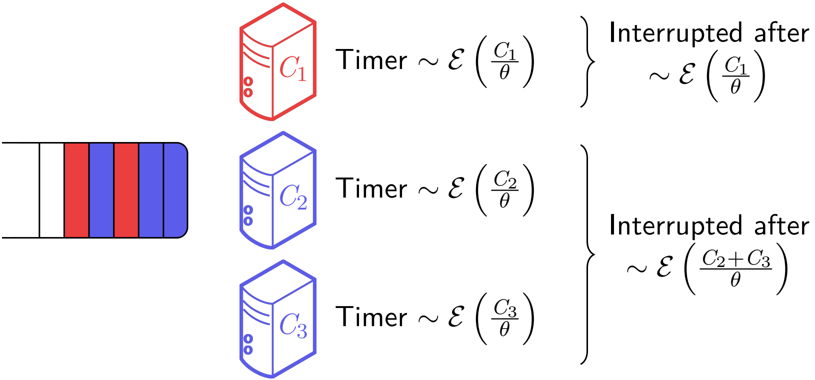


Timer $\sim \mathcal{E}\left(\frac{C_1}{\theta}\right)$

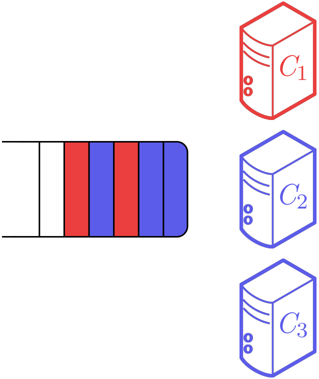
Timer $\sim \mathcal{E}\left(\frac{C_2}{\theta}\right)$

Timer $\sim \mathcal{E}\left(\frac{C_3}{\theta}\right)$

Computer cluster



Computer cluster



Timer $\sim \mathcal{E}\left(\frac{C_1}{\theta}\right)$ — Interrupted after $\sim \mathcal{E}\left(\frac{C_1}{\theta}\right)$

Timer $\sim \mathcal{E}\left(\frac{C_2}{\theta}\right)$

Timer $\sim \mathcal{E}\left(\frac{C_3}{\theta}\right)$

Interrupted after $\sim \mathcal{E}\left(\frac{C_2+C_3}{\theta}\right)$

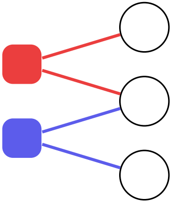
When $m \to \infty$, resources allocated according to balanced fairness

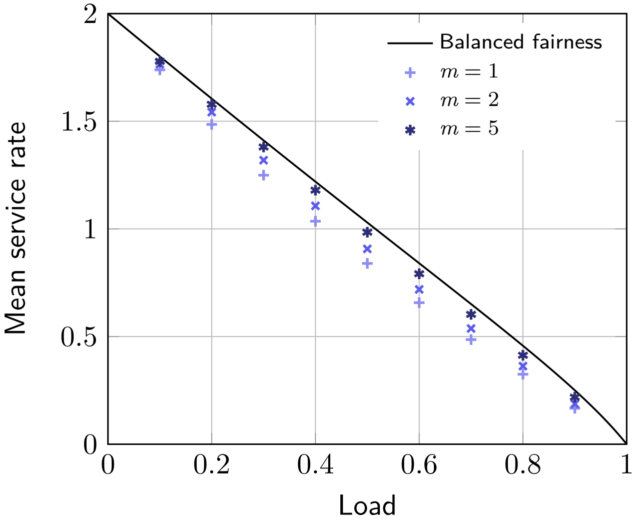
Numerical results

- Performance metric:
  Mean service rate seen by class-i jobs

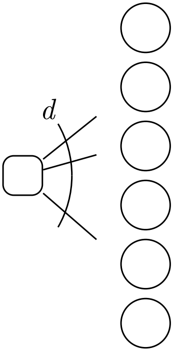
- Hyperexponential job size distribution
  $\rightarrow$ $\sim \mathcal{E}(1/5)$ with probability $1/6$
  $\rightarrow$ $\sim \mathcal{E}(5)$ with probability $5/6$

Numerical results: Shared pool





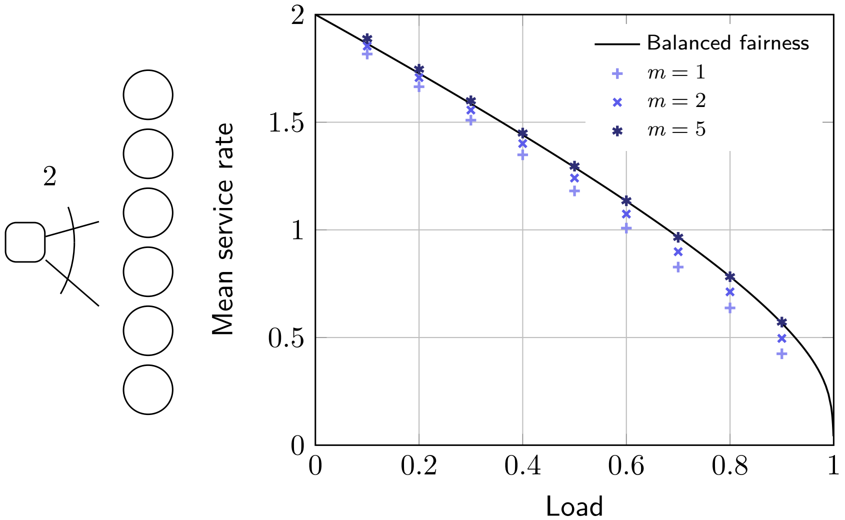
Numerical results: Random assignment



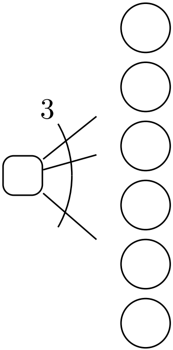
- d servers chosen uniformly and independently at random
- By (Gardner et al., 2016),

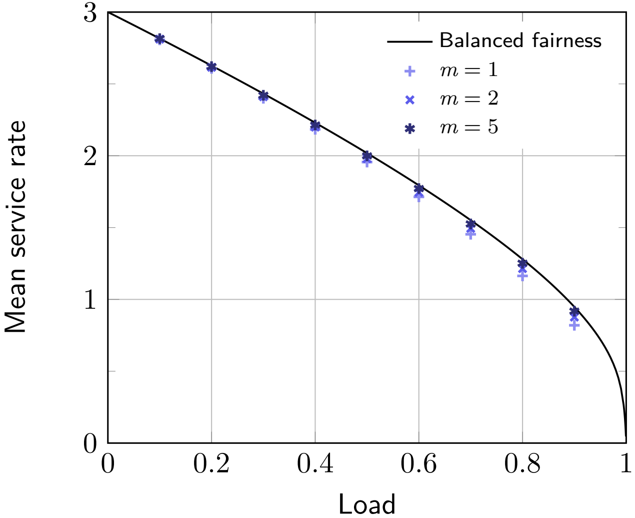
$$\frac{1}{\gamma} = \sum_{j=d}^{S} \frac{1}{S\mu \frac{\binom{S-1}{d-1}}{\binom{j-1}{d-1}} - S\lambda}$$

Numerical results: $S = 100$, $d = 2$



Numerical results: $S = 100$, $d = 3$





Conclusion

- Multi-server queues with parallel processing
  - Sequential version of a class of Whittle networks
  - Stable whenever each set of classes can handle its own load
  - Closed-form expressions for the performance metrics
- Scheduling algorithm in computer clusters
  - Service interruptions implemented by exponential timers
  - Insensitive resource allocation
- Future works
  - Generalize these results to Order Independent queues
  - Assert robustness of the algorithm

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