Synthesizing Decentralized Components from a Variant of Live Sequence Charts

Dirk Fahland¹ and Amir Kantor²

¹Eindhoven University of Technology, the Netherlands
²Weizmann Institute of Science, Rehovot, Israel
d.fahland@tue.nl, amir.kantor@weizmann.ac.il

Keywords: Live Sequence Charts, Scenarios, Decentralized Synthesis, Petri Nets, Partially Ordered Runs

Abstract: Live sequence charts (LSC) is a visual, executable, language for the modeling of reactive systems. Each chart depicts an inter-object scenario arising in the modeled system, partitioned into two: a monitored prechart, and a main chart. Despite the intuitive use of the language, complications arise when one wants to implement an LSC specification with decentralized components. In this paper, we introduce a variant of LSC, called distributed LSC (dLSC), which is targeted for the modeling and synthesis of decentralized systems, composed of several interacting components. While LSCs are commonly interpreted in terms of an interleaved execution of the scenarios in a sequential run, dLSCs employ partially ordered runs. We investigate the expressive power of dLSC compared to an established model of concurrent systems, namely, Petri nets, and show that dLSCs are, computationally, strictly more expressive than low-level Petri nets and subsumed by higher-level Petri nets. Specifically, we present an algorithm that synthesizes, given a dLSC specification, an equivalent token history net, which can serve as an executable implementation of the specification. Most importantly, the implementation is decentralized — components can be automatically extracted from the net. The synthesis of Petri-net components from a dLSC specification is supported by a tool.

1 INTRODUCTION

The visual language of live sequence charts (LSC) (Harel and Marelly, 2003), introduced in (Damm and Harel, 2001), is an intuitive way of modeling reactive systems with scenarios. It originated from the scenario-based formalism of message sequence charts (MSC) (ITU, 1996). Scenarios visually describe interactions among components and objects of the system. This inter-object behavior is aligned along time-lines, corresponding explicitly to the runs of the modeled system. In that respect, scenarios are dual to the intra-object perspective taken in traditional system-models such as statecharts (Harel, 1987) and Petri nets (Reisig, 1985). In the latter, the structure of the model is in accordance with object (or component) boundaries, while the notion of time is implicit.

Scenarios have been found to be very useful in industry to describe system behavior, particularly at earlier stages of system design. System-models, in contrast, which are sometimes harder to devise, are useful as blueprints for implementing the system in hardware or software. Consequently, a notable challenge is to synthesize from a specification in the form of a set of scenarios, a system-model, called an implementation of the specification, which behaves as specified in the scenarios.

In this paper, we address scenario-based modeling of concurrent systems (Ben-Ari, 2006), and the problem of synthesizing such systems from the specifications. Concurrent systems involve several inter-related components (or, processes) that are executed simultaneously, so that control is decentralized among the components. Such systems are very common. Concurrency may be motivated by pragmatic considerations; e.g., to boost system performance, especially in the presence of a decentralized architecture such as a multi-core processor or a cluster of computers. Sometimes, however, concurrency is dictated by the underlying architecture. Examples for such systems are web-services executed over the Internet, and embedded systems composed of autonomous controllers. Moreover, real-life processes, e.g., business processes, carried out by several autonomous persons or units, can be modeled and analyzed as concurrent systems.

Scenarios, which present interactions between components in a partially ordered structure, can naturally describe executions of concurrent systems. In
fact, MSCs are extensively used to describe sample interactions in concurrent systems and distributed protocols. Yet, MSCs are essentially too weak to capture the logic that underlies most systems. LSC enriches the scenarios of MSC, mainly by being multi-modal, and makes them expressive enough to become a fully-fledged model for the system, expressively comparable to intra-object behavioral models.

However, the language of LSC, in its present form, is not well suited for the modeling of concurrent, decentralized systems. First, play-out (Harel and Marelly, 2003), the executable semantics of LSC, defines a central controller that implements the system as a whole. Moreover, regardless of how play-out is defined, it is shown in (Bontemps and Schobbens, 2007) that without additional coordination, some LSC specifications cannot be distributed into components. As we discuss in Sect. 2, the standard interpretation of LSC (Harel and Marelly, 2003) results in implicit dependencies between the different parts of a scenario, which arise throughout any typical specification. As long as the system is implemented as a single controller, this raises no difficulty. However, in decentralized architectures, such dependencies require more interaction between the components than specified.

If we were to use LSC, or any other formalism, for specifying concurrent systems, the behavior that can be specified in that formalism must be such that it can be exhibited by decentralized components. In this context, it is significant to impose a restriction on the components, that they coordinate and interact with each other merely as described in the specification. Otherwise, components are not as autonomous, and the system is less decentralized than intended. In typical LSC specifications, the amount of additional interaction required is significant, and would result in a major efficiency overhead.

With the intention to support the modeling of concurrent systems, we introduce a variation on the semantics of LSC. It is applied on a central fragment of the language, which includes scenarios partitioned into a prechart and a main chart (see Sect. 2). Instead of the traditional interpretation, presented in terms of interleaved sequential runs, we interpret LSC specifications on the basis of partially ordered runs (Pratt, 1986); i.e., traces of executions in which events are partially ordered. In such a semantic domain, also known as a true-concurrency semantic domain, we adopt LSC’s prechart/main-chart distinction. As runs are partially ordered, they convey more information than interleaved runs, regarding the causal dependencies between the events. Here, as both scenarios and runs are partially ordered, a fragment of a scenario can be identified with a matching sub-structure in the run.

Changing the semantic domain results in a simple variant of the language, which we refer to as distributed live sequence charts (dLSC). dLSC avoids implicit dependencies between separate parts of a scenario, and is thus, we believe, well suited for the modeling of concurrent systems. Moreover, as partial order runs directly correspond to the visual structure of charts, our interpretation is simple and comprehensible, and has a rigorous mathematical basis. We demonstrate the language and its use with a case study.

We investigate the expressive power of dLSC with respect to a common model of concurrent systems, namely, Petri nets (Reisig, 1985). We show that dLSC specifications are, effectively, strictly more expressive than low-level Petri nets in the form of place/transition nets (Reisig, 1985). However, they do not exceed the expressive power of high-level nets; dLSC specifications are subsumed by the class of token history nets (van Hee et al., 2008).

We present an algorithm that synthesizes, for any given dLSC specification $S$, an equivalent token history net $N_S$. A token history net, being a particular kind of a coloured Petri net (Jensen, 1987), is an executable model, and thus may serve as an implementation of the specification. Moreover, and most importantly, the implementation is decentralized — the components specified in $S$ can be extracted from the resulting net $N_S$, and, for a large class of specifications, no additional interaction between the components is involved. The synthesis of Petri-net components from a dLSC specification is supported by a prototype tool.

The paper is structured as follows. In Sect. 2, the semantics of LSC is discussed more closely. In Sect. 3, we introduce the variant of distributed LSC through an example, whereas a formal representation of the formalism and its semantics is given in Appx. A.1. In Sect. 4, we investigate the expressive power of dLSC. Our technique to synthesize system-models from dLSC specifications and to extract decentralized components from them is presented in Sect. 5 and 6, as well as our prototype tool. We discuss related work in Sect. 7, and conclude in Sect. 8.

## 2 FROM LSCs TO DISTRIBUTED LSCs

In Fig. 1a, we illustrate a live sequence chart $L_a$. In the chart, there are three vertical lines, called lifelines, which correspond to three objects: $A$, $B$, and $C$. The interactions between the objects are depicted by four arrows, labeled by $a$, $b$, $c$, and $d$, which designate events or messages. Time passes along lifelines from top to bottom, which determines the order between
the events (namely, a through d in that exact order). Events in LSCs are, in general, partially ordered. $L_{\omega}$ is divided into two: a prechart, depicted inside a dashed hexagon (containing event a), and a main chart, depicted inside a solid rectangle (containing events b through d). The prechart and the main chart are of two complementary modalities: monitored versus execute. The prechart is monitored; i.e., it is matched at run-time against the events that are executed, but does not yield new behavior. The main chart, in contrast, supplements runs with new behavior. If and when the prechart is met, the main chart is enabled and thus executed. Accordingly, in $L_{\omega}$, if and when event a occurs, events b, c, and d are executed.

There is another multi-modal distinction in the language of LSC, between cold behaviors, which may happen in the system (possible), and hot behaviors, which must happen (mandatory). In $L_{\omega}$, all events are cold (possible), which is designated by blue dashed arrows, and thus may be discarded in the presence of other, conflicting, alternatives. The following observations are independent of the hot/cold distinction.

The common semantics of LSC (see, e.g., (Harel and Marelly, 2003)) is based on an interleaved execution of LSCs; i.e., a sequential run is constructed from the interleaving of partial order scenarios. An LSC is one consolidated structure, in the following sense: if any of the events that appear in the chart happens to occur out of the order prescribed by the chart, the scenario is violated, and should be aborted. Consider, for example, the chart $L_{\omega}$ presented in Fig. 1a, describing interactions between three objects, A, B, and C. If event a occurs for some reason (perhaps due to some other chart) after a, b, and c have all occurred, but before d, then C should abort the scenario without executing d. In order to achieve this kind of behavior, C must be aware of occurrences of a. Thus, if the system is to be implemented by decentralized components, C must be notified of the executions of a (even those coming from outside the present chart) one way or another. There are many such implicit dependencies in $L_{\omega}$ alone. E.g., such a dependency arises also between b and d, so that C must be aware of executions of b, and A must be aware of executions of d.

If the system is to be implemented in a decentralized architecture, such dependencies introduce significant complications. They would require additional unspecified interactions between the components, resulting in communication overheads and excessive run-time synchronization among the components. In this paper we establish the use of LSCs, and specifically the language’s prechart/main-chart distinction, in a semantic domain that is more suited to the modeling of concurrent systems. The resulting formalism is referred to as distributed LSC (dLSC). In this paper we address a basic, central, fragment of the language of LSC. We consider charts, each partitioned into a prechart and a main chart, containing cold events.

3 THE VISUAL FORMALISM OF DISTRIBUTED LSC

We introduce distributed LSCs in the context of an example, which involves concurrently operating components. We model the behavior of an emergency management procedure. The procedure involves one or more medics, providing first-aid treatment, a clinic, and an Emergency Management System (EMS), which keeps track of pending emergencies and mediates between the medics and the clinic.

3.1 Scenarios

A dLSC specification is a finite set of dLSCs (and an initial run) which together describe the system’s behavior. Fig. 2c shows an illustrative dLSC of the procedure, denoted by $L_3$. A dLSC is a partial ordering of events; events are drawn as rectangles, and the ordering of events is indicated by arrows. The horizontal dashed line divides $L_3$ into a monitored prechart (denoting the precondition that enables $L_3$) and a main chart (denoting the behavior contributed by $L_3$, which is to be executed once the prechart is met). The vertical lifelines in $L_3$ are used to graphically align events of the same component but have no formal meaning.

The prechart of $L_3$ consists of two unordered events, labeled EMS.alert and M.ready. Throughout the specification, the events of the ith medic are prefixed by $M_i$ (for different concrete values of i), those of the clinic are prefixed by C, and those of the EMS by EMS. Event EMS.alert represents a notification from the EMS of a pending emergency. Event M.ready designates a notification by the medic that he has become ready to handle emergencies. If and when the EMS notifies of a pending emergency, and the medic

Figure 1: A live sequence chart $L_{\omega}$
which the medic is available again (and the patient need not be brought to the clinic). dLSCs dLSC2
and dLSC3 have the same prechart. As we consider all events to be cold (possible), we understand such scenarios as alternatives: whenever the prechart of dLSC2 and dLSC3 is met, the execution continues according to either dLSC2 or dLSC3.

dLSC dLSC1 of Fig. 2a captures the arrival of emergency calls to the EMS. Whenever the EMS is ready (EMS.ready), a new emergency may arrive, resulting in two independent events: the EMS alerts the medics of a pending emergency (EMS.alert), and the EMS becomes ready again to receive more emergencies (EMS.ready). Finally, a specification contains an initial run that describes how the procedure begins. It is depicted in Fig. 2e and is denoted by R0. In our example, R0 includes four unordered (independent) events: the events EMS.ready, two events labeled M1.ready and M2.ready (assuming the process involves two medics; any number of medics is supported), and the event C.ready.

3.2 Semantics

Syntactically, a dLSCs is just an LSC drawn in a slightly more abstract form. For instance, LSC Lα of Fig. 1a can be represented as in Fig. 1b. Where dLSCs and LSCs actually differ is in their interpretation. Instead of LSC’s interleaved semantics, we interpret dLSCs on the basis of Pratt’s partially ordered runs (Pratt, 1986), a common framework to describe the behavior of concurrent systems.

Partially ordered runs. Fig. 3a shows a partially ordered run ρ1. It consists of 9 events (drawn as rectangles) that are labeled and partially ordered according to the directed arcs (the dashed vertical lines align events graphically but have no formal meaning). A partially ordered run captures the causal dependencies between events — an event occurs after all its predecessors have occurred. For instance, in Fig. 3a, events EMS.ready, M1.ready, M2.ready and C.ready can all occur in the beginning, i.e., they are mutually independent. Once EMS.ready occurred, EMS.alert and the second EMS.ready event occur; M1.go can only occur after both, M1.ready and EMS.alert have occurred.

A partially ordered run ρ corresponds to a set of sequential runs, each being an interleaving of the events in ρ that is consistent with the partial order in ρ. Such an interleaving corresponds to what a global observer overlooking the execution might see.

Scenarios describe partially ordered runs. As individual scenarios are themselves fragments of partially ordered runs, the latter seems a natural candidate for the semantic domain. When executions are represented as partially ordered runs, the ordering of events in a scenario directly carries over to the runs, and individu-
Figure 3: Runs of the emergency management procedure

ual scenarios can be recognized inside the run. This perspective suggests an alternative way to interpret LSCs.

The behavior induced by a dLSC specification $S$ may be briefly described as follows. The specification is executed starting with the initial, partially ordered, run ($R_0$ in our example). Whenever the run ends with a pattern that matches the prechart of some dLSC $L$ in $S$, the events in the main chart of $L$ are locally concatenated. With such concatenations, partially ordered runs are augmented, possibly ad infinitum. The exact formal semantics are given in Appx A.1. In the following, we illustrate this semantics by our running example.

The partially ordered run in Fig. 3a, which we denote by $\rho_1$, is an example of an execution of the emergency management procedure. It is obtained as follows. Starting with the initial run $R_0$, the prechart of $L_1$ is met, and so its main chart is concatenated. This results, in particular, in the creation of the event EMS.alert. Then, the precharts of both $L_2$ and $L_3$ are met, so either one may be concatenated. $\rho_1$ is the result of concatenating $L_2$. The other possibility, of concatenating $L_3$, appears in run $\rho_2$ of Fig. 3b. Runs $\rho_1$ and $\rho_2$ are alternatives. In $\rho_2$, the concatenation of $L_3$ results, in particular, in the occurrence of the event C.enroll. Then, the prechart of $L_4$ is met, and so the main chart of $L_4$ is also concatenated.

A slightly more involved execution is $\rho_3$ of Fig. 3c. It contains two EMS.alert events. The first alert is handled by $M_1$ according to dLSC $L_2$, and the second alert is handled by $M_2$ according to dLSCs $L_3$ and $L_4$. Runs $\rho_1$, $\rho_2$, and $\rho_3$ can be continued, possibly ad infinitum. Note that the activities of the two medics in $\rho_3$ are unordered, reflecting the fact that the two medics operate independently.

3.3 The Extended Example

We incrementally extend the emergency management procedure with three additional dLSCs. $L_5$, depicted in Fig. 4a, describes another alternative to $L_2$ and $L_3$: a medic reaching the patient may realize that the clinic needs to prepare for the incoming patient. The medic notifies the EMS of the incoming patient ($M_i$.notify), which in turn notifies the clinic (EMS.notify). According to dLSC $L_6$ of Fig. 4b, the clinic prepares for the arrival of the patient (C.prepare), and then waits for the patient (C.wait4), concurrently to the other duties of the clinic (due to C.ready). After the patient has enrolled in the clinic, he is treated according to dLSC $L_7$ (see Fig. 4c).

An execution of the extended specifications is depicted in Fig. 5. The run, denoted $\rho_4$, is similar to $\rho_3$ of Fig. 3c, but the second medical emergency is treated according to $L_5$. After the concatenation of $L_5$ and $L_6$ the prechart of $L_7$ is matched, while the prechart of $L_4$ is not; only the main chart of $L_7$ can be added, after the events C.enroll and C.wait4. dLSC $L_7$ illustrates the expressive power of precharts to describe behavior across components. dLSC $L_4$ and $L_7$ both include the event C.enroll in their precharts, but it is preceded by different events (namely, $M_i$.treat in $L_4$ and $M_i$.treat in $L_7$). Therefore, the precharts reflect different sit-
Figure 4: Extending the emergency management procedure

Figure 5: A run \( \rho_4 \) of the extended emergency management procedure

Figure 6: Translating a place/transition net into a dLSC specification

4 EXPRESSIVE POWER

We just introduced dLSC, which interprets the core concepts of LSC in the context of partially ordered runs. In this section, we discuss whether this core language and interpretation are sufficiently expressive to describe decentralized systems.

Distributed LSCs subsume Petri nets. Distributed LSCs can be seen to subsume low-level Petri nets in the form of place/transition nets (PTN) (Reisig, 1985; Peterson, 1977). PTNs are an abstract model for the flow of control and information in systems, particularly concurrent and decentralized systems.

A PTN consists of places \( P \) (drawn as circles) and transitions \( T \) (drawn as rectangles) that are connected by arcs from places to transitions and from transitions to places; see, for example, PTN \( N_b \) depicted in Fig. 6a. The global state of the net is given by a marking which puts in each place a nonnegative number of tokens; a PTN has a dedicated initial marking. Given a marking, a transition \( t \) is enabled if each place with an arc to \( t \) has a token. If \( t \) is enabled, it may fire, which results in a new marking obtained by removing a token from each place with an arc going to \( t \) and putting a token on each place with an arc coming from \( t \). These notions give rise to both an interleaved semantics, presented in terms of sequential runs, and a true-concurrency semantics in terms of partially ordered runs that is consistent with the interleaved semantics (Goltz and Reisig, 1983). The partially ordered runs of a Petri net can be constructed by local continuations — each firing of a transition is recorded as a local continuation. Figure 6c shows a partially ordered run of PTN \( N_b \).
of Fig. 6a as follows; transition $t_1$ occurred, consuming a token from $p_3$ and producing a token on $p_2$; $t_3$ occurred concurrently to $t_1$, consuming from $p_5$ and producing on $p_4$; transition $t_2$ occurred after $t_1$ and $t_3$, consuming from $p_2$, $p_3$ and producing on $p_3$.

Next, we show that dLSC are expressive enough to specify any place/transition Petri net. Given a PTN $N$, one can construct an equivalent dLSC specification $S_N$. We take $\Sigma := T \cup P$ to be the set of actions in our specification, which includes both transitions and places. Places can be considered as auxiliary actions, and can be abstracted away from the runs induced by the specification, in case one is only interested in the events that are due to the firing of transitions.

For each transition $t \in T$, we construct a dLSC $L_t$ as follows (see Fig. 6b, illustrating the dLSC corresponding to transition $t_2$ of $N_b$). The prechart of $L_t$ contains the input places of $t$ as events. There is no ordering between the events in the prechart. The main chart of $L_t$ begins with the event $t$, after which the output places of $t$ are included with no ordering between them. The dLSC specification corresponding to the net $N$ contains one dLSC $L_t$ for each transition $t$ of $N$, and an initial run $R_0$, where $R_0$ contains for each place $p$ of $N$ as many $p$-labeled events as there are tokens on $p$ in the initial marking, with no ordering between the events. This construction also applies to place/transition nets with arc weights, by duplicating events representing places according to the weights.

It can be shown that the set of runs of $S_N$ is isomorphic to the set of partial order Petri-net runs of $N$. The idea is to represent the latter on the basis of local continuations. Each continuation rule for constructing the Petri-net runs, corresponds to a dLSC in $S_N$. Fig. 6c illustrates the Petri-net run of $N_b$, starting from the initial marking, while Fig. 6d depicts the corresponding run of the dLSC specification $S_N$.

Strictly more expressive than Petri nets. The converse proposition, that each dLSC specification can be translated into an equivalent PTN, does not hold. Intuitively, PTNs cannot mimic the enabling condition expressed by precharts with a complex structure. The enabling of a Petri net transition depends only on the availability of tokens in its preplaces and nothing else; the enabling of a dLSC can depend on several past events and their causal ordering. The formal proof that establishes the greater expressive power of dLSC compared to PTN is given in (Fahland, 2010). There, it is shown that any instance of Post’s correspondence problem (PCP) can be expressed as a dLSC specification, such that a particular event occurs if and only if the PCP instance has a solution. In PTN, the problem of deciding whether a particular event can occur is decidable, whereas PCP is undecidable. Therefore, there is no algorithm to translate dLSC specifications into equivalent PTNs.

5 SYNTHESIZING SYSTEMS

Section 4 shows that distributed LSC, our interpretation of LSC in the context of partially ordered runs, allows to specify the behavior of a large class of concurrent systems. In the remainder of the paper, we address the following problem, which may be referred to as the decentralized synthesis problem: given a dLSC specification $S$ (i.e., a set of dLSCs, in which events are assigned to components, and an initial run), synthesize an implementation consisting of decentralized components, in a suitable system-model formalism, which behave and interact exactly as specified in $S$.

Section 4 shows that this problem is not trivial, and that the class of simple place/transition nets is not expressive enough to capture the behavior specified in dLSC specifications. In order to solve the decentralized synthesis problem, we use a slight extension of place/transitions nets, called token history nets (van Hee et al., 2008), to represent the synthesized implementation. In the present section, we show how to effectively synthesize from a given dLSC specification $S$ an equivalent token history net $N_S$. Then, individual components can be easily extracted from $N_S$, which is discussed in Sect. 6.

The synthesis of $N_S$ is carried out as follows. Events of $S$ are translated into transitions in $N_S$, and the partial order between them is enforced in $N_S$ through Petri-net places. To capture that the occurrence of some event of $S$ depends on its preceding events, we use the fact that tokens in $N_S$ record their own history, in terms of the transitions that they have passed. A transition in $N_S$ will only be enabled by tokens with the correct history. We first present the class of token history nets, and then define the synthesis of $N_S$ from $S$.

5.1 Token History Nets

This part gives an informal introduction to Token History Nets, the formal definitions are given in Appx. A.2. A token history Petri net (THPN) (Van Hee et al., 2007; van Hee et al., 2008) is a Petri net in which transition are labeled with actions $\Sigma$ or with $\tau \notin \Sigma$; $\Sigma$ are observable actions (which will represent the actions in a dLSC specification), while $\tau$ is a silent (or, unobservable) action. The main difference to place/transition nets is that each token of a THPN is a partially ordered run as discussed in Sect. 3.2 representing the history of transition firings that have led it to its current place. A
firing of a transition extends the histories of the tokens involved.

Figure 7a shows a token history net. As usual, a circle represents a place, a rectangle represents a transition, and transition labels are inscribed. Moreover, each transition has a guard in form of a token history (shown for the transitions that go by the names \(L_2\) to \(L_5\), and \(L_7\)). Intuitively, a transition is only enabled if the token histories in its pre-places together end with the token history in the guard.

We illustrate the semantics of THPNs with a partially ordered run (Goltz and Reisig, 1983) \(\rho\) of the THPN \(N\) of Fig. 7a. Run \(\rho\) is shown in Fig. 7b as an acyclic labeled Petri net: each place of \(\rho\) (called a condition) with label \(p\) represents a token history on the place \(p\); a transition \(e\) of \(\rho\) (called an event), with label \(t\), represents a firing of transition \(t\) of \(N\); the pre-places (post-places) of \(e\) represent the token histories consumed (produced) by \(t\).

For instance, in Fig. 7b, condition \(b_3\) denotes that the place \(E.ready\) is marked with history \(h_0\) consisting only of event \(E.ready\). In this situation, transition \(L_1\) is enabled. Event \(e_4\) denotes the firing of \(L_1\), which consumes \(h_0\) from \(E.ready\) and produces \(h_1\) (\(h_0\) extended with the occurrence of the silent transition \(L_1\)) on both \(p_3\) and \(p_4\), as represented by conditions \(b_6\) and \(b_7\) in \(\rho\). Event \(e_9\) denotes the firing of the transition labeled \(E.alert\) (the one consuming from \(p_4\)), which consumes \(h_1\) from \(p_4\), and produces \(h_3\) on place \(E.alert\) as represented by condition \(b_9\). The run of Fig. 7b shows how the token histories are built up event by event, eventually joining several token histories into one at event \(e_6\). Note that transition \(L_2\) is only enabled because the union of histories \(h_3\) and \(h_7\) ends with the guard of \(L_2\). Guards in a THPN can also be more complex such as the guard of \(L_7\) which requires token histories on \(C.enroll\) and \(C.wait\) to have a joint event \(M_1\), notify.

### 5.2 Translating Specifications into Token History Nets

In the execution of a THPN, each token history records the preceding events as a partially ordered run. This allows us to capture the semantics of dLSC specifications with token history nets. Fig. 7a, for instance, depicts the result of the translation of the specification of Sect. 3 into a THPN (to avoid cluttering the figures, we show only one of the medics). The formal translation is included in Appx. A.3.

#### Translating the specification.

We translate a dLSC specification \(\Sigma = \langle D, R_0\rangle\) over actions \(\Sigma\) (where \(D\) is a set of dLSCs and \(R_0\) is the initial run) into an equivalent THPN \(N_\Sigma\) over \(\Sigma\). We first translate each chart \(L \in D\) into a net \(N_L\), and then, compose the resulting nets to form the net \(N_S\) of the entire specification. The different \(N_L\)'s are connected via shared places: each maximal main-chart event labeled \(a\) of some chart \(L \in D\) defines a shared place \(p_a\), on which \(N_L\) produces. For a chart \(L' \in D\), in which an event labeled \(a\) appears maximal in the prechart, \(N_{L'}\) will consume from \(p_a\).

#### Translating individual charts.

Each dLSC \(L\) in the specification induces a net \(N_L\). For each main-chart event \(e\) of \(L\), \(N_L\) contains a transition \(t_e\) that gets the same label as \(e\). The partial order of \(L\)'s main chart is encoded by places. In addition, each transition \(t_e\) of \(N_L\) gets a guard that ensures that \(t_e\) is only enabled if the token history produced by \(t_e\) ends with the history of \(e\) in \(L\), i.e., the events preceding \(e\) in \(L\). The initial run \(R_0\) is translated into a net \(N_{R_0}\) in the same way.

In Fig. 7a, the result of translating the main charts of the specification of Sect. 3 as described above is shown inside the shaded boxes. For instance, considering dLSC \(L_2\), the net contains a transition labeled \(M_1.go\) for the minimal event in the main chart of \(L_2\), preceded by the activation place \(p_9\). The last event in \(L_2\), labeled \(M_1.ready\), produces on the shared place \(M_1.ready\). The subnet \(N_{R_0}\) of the initial run is scattered throughout between the other subnets; its activation places are \(p_0\), \(p_1\), and \(p_2\).

The transitions of \(N_L\) representing the minimal events in the main chart of \(L\) shall only be enabled when all the maximal events in \(L\)'s prechart have occurred. We formalize this by a main-chart activation transition \(t_1\) with label \(\tau\) (unobservable); \(t_1\) consumes from the shared places that contain messages from \(L\)'s prechart events and produces on places that enable the minimal main chart events of \(L\). In addition, \(t_L\) has a guard that enables \(t_L\) only if the prechart of \(L\) has occurred. A firing of \(t_L\) will not be visible in the resulting token history as \(t_1\) has a label \(\tau\). E.g., in Fig. 7a, the \(\tau\) transition \(L_2\) is the activation transition of dLSC \(L_2\) of Fig. 2b. When checking whether the token histories consumed by a transition \(t\) satisfy the guard of \(t\), \(\tau\)-labeled events in the tokens are ignored.

The synthesized net \(N_S\) exhibits the same behavior that is specified in \(S\). More precisely, the partial order runs of \(N_S\) are the same as (i.e., isomorphic to) those of \(S\), after the events of \(\tau\)-labeled transitions are abstracted away from the net’s runs. That is, the specified behavior is refined by unobservable actions (more on this in Sect. 6).

As specified above, token histories can grow indefinitely. However, guards of transitions only consider the more recent events. Thus, token histories can be bounded by the longest chart in the specification, by truncating (e.g., in each transition) older events.
6 EXTRACTING COMPONENTS

In Sect. 5, we introduced a technique to synthesize from a dLSC specification $S$, a THPN $N_S$ with the same behavior. In this section we proceed and extract decentralized Petri-net components from $N_S$. This would complete the path from a decentralized scenario-based model — namely, a dLSC specification $S$ — to Petri-net models of the components that implement it.

6.1 Components

As in our running example, we assume that in the specification each action is performed by a particular component, which is denoted explicitly. Specifically, we assume a finite set of components $C$, such that each action of $S$ is of the form $c.e$ for some component $c \in C$ and an event name $e$.

Each transition $t$ in $N_S$ is then either labeled by an action $c.e$, or it is a $\tau$-labeled transition that captures the activation of the main chart of some dLSC $L$ (denoted by $t_L$ in Sect. 5.2). In the former case, $t$ is associated with the component $c$ that performs the underlying action. In the latter case, $t$ must be assigned to a component; this is an important matter that we address in Sect. 6.2. Assigning transitions to components naturally induces a decomposition of the net $N_S$ into Petri-net components.

For each $c \in C$, a Petri-net component $N_c$ is defined to be the subnet of $N_S$ containing the transitions assigned to $c$, denoted by $T_c$, the places $P_c$ that are directly connected to the transitions in $T_c$, and the arcs between $T_c$ and $P_c$ as appears in $N_S$. This standard construction is formalized in Appx. A.4. A place $p$ belonging to more than one component is an interface place; otherwise, $p$ is internal.

According to such decomposition, when the components are put together, they yield the original net $N_S$. Therefore, when executed, the components exhibit precisely the behavior of the net $N_S$. As discussed in Sect. 5, this behavior matches that prescribed in dLSC specification $S$.

Considering our running example, actions are prefixed by a component name: either E (the EMS), M, (the i-th medic), or C (the clinic). Extracting components from the synthesized net of Fig. 7a as described above yields the components shown in Fig. 8. This figure was obtained using our tool SAM, which is described in the following.

6.2 Interactions between Components

When considering decentralized synthesis from specifications, the latter must impose restrictions on how components must interact with each other. Without any limitation in that respect, one could construct components that interact arbitrarily. This would undermine...
Unobservable actions. Compare the partially ordered runs of Fig. 9, which include \( \tau \)-labeled events, with the corresponding runs of Fig. 10 in which the \( \tau \)-labeled events are abstracted away and the causal dependencies between the observable events remain intact. The runs in Fig. 10, which record only observable events, correspond to specified behavior, while the runs of Fig. 9 correspond to the runs of an implementation containing also unobservable transitions. In the runs, each event, including the unobservable events, is associated with a particular component.

A direct causal dependency between events of different components gives rise to an interaction between the components. E.g., in Fig. 10a, as event \( c_1.e \) directly causes event \( c_3.f \), there is an underlying interaction between the components. Depending on how \( \tau \)-labeled actions are performed, the interaction scheme may change: in Fig. 9a, \( c_1 \) and \( c_3 \) no longer interact with each other; rather, through the \( \tau \)-labeled event of component \( c_2 \), \( c_1 \) interacts with \( c_2 \) and \( c_2 \) with \( c_3 \). This is exactly the situation that must be avoided when a decentralized implementation contains unobservable actions that refine the specified behavior.

Figures 9b and 9c show situations in which the refined behavior of the implementation presents the same interaction scheme as in the specified behavior. In Fig. 9b and 10b, all events belong to the same component, so no interaction is present. In Fig. 9c, there is an interaction between \( c_1 \) and \( c_2 \), which is also the case in Fig. 10c (multiple arrows from one event in \( c_1 \) to multiple events in the same component \( c_2 \) count as one interaction). These two cases illustrate sufficient conditions in which unobservable events do not change the interaction scheme: a \( \tau \)-labeled event \( x \) is termed pre-internal (post-internal) if all direct successor (predecessors) events of \( x \) are performed by the component performing \( x \); \( x \) is internal if it is pre- and post-internal. In Fig. 9b, the \( \tau \)-labeled event is internal, in Fig. 9c it is pre-internal, and in Fig. 9a it is neither.

In the runs of the implementation, an unobservable event that is pre- or post-internal does not change the interaction scheme between the components compared to the specification. Thus, when assigning the \( \tau \)-labeled transitions of \( N_S \) to components, we have to make sure they only yield pre- or post-internal events. A Petri-net transition \( t \) yields only pre-internal events if all post-places (having an arc from \( t \)) are internal places, since the succeeding transitions belong to the same component as \( t \); similarly, \( t \) yields only post-internal events if all pre-places are internal.

Assigning activation transitions. There is a class of specifications for which there is a natural way to assign activation transitions to components. A dLSC \( L \) is called local choice if all the minimal events in its main chart are of the same component \( c \in C \). Intuitively, in this case, only component \( c \) is involved when the main chart begins, and thus the choice for an activation of the chart can be made locally in component \( c \). For a
local choice dLSC \( L \), the activation transition of \( L \) is assigned to component \( c \) as well. A specification \( S \) is said to be local choice if all the dLSCs in \( S \) are local choice. E.g., the specification of our running example, as one may easily verify, is local choice.

In local choice specifications, when activation transitions are assigned to components as indicated above, any occurrence of a \( \tau \)-labeled action in the run is pre-internal. This can be deduced from the structure of \( N_S \), as the post-places of each activation transition are internal. Therefore, the \( \tau \)-labeled transitions in \( N_S \) do not change the interaction scheme between the components compared to the specification.

As for non-local choice charts, activation transitions need to be explicitly assigned by the modeler. These can be made explicit by refining the specification to become local choice.

**Tool support.** Our technique for synthesizing Petri-net components from dLSC specifications is implemented in a prototype tool called SAM. The tool takes as input a dLSC specification in a simple textual syntax, describing each dLSC’s prechart and main chart as a partial order of events. Additionally, components can be specified as sets of event names. SAM produces a token history net with extracted components as a CPN Tools (Jensen et al., 2007; Ratzer et al., 2003) model. CPN Tools implements the general class of coloured Petri nets (Jensen, 1987), which allow to define datatypes for token histories, and to represent the firing rule of a THPN with operations on tokens. This allows analyzing the resulting components using the full grown simulation functionality of CPN Tools. Fig. 11 shows the components appearing in Fig. 8 within CPN Tools. Our implementation currently handles only specifications in which the maximal events in a prechart are labeled differently. SAM is available at [http://www.win.tue.nl/~djalhland/tools/sam/](http://www.win.tue.nl/~djalhland/tools/sam/).

### 7 RELATED WORK

As discussed in Sect. 2, dLSC builds on ideas from the language of LSC (Harel and Marelly, 2003), and implements them in a semantic domain that is more directly related to concurrent systems, based on partially ordered runs. The change in the semantic domain allows to identify scenarios with patterns that explicitly appear in the constructed run. This contrasts with LSC, which identifies scenarios with their interleavings.

In LSC, the concurrent execution of two charts depends on the identity of their events; that is, whether one can be executed without violating the other. If one violates the other, the charts become alternatives (in case of cold main chart events). In dLSC, in contrast, causality is recorded in the run: two charts are executed concurrently if they are enabled at two causally independent parts of the run, and they are alternatives if they are enabled at overlapping (or identical) parts of the run. Thus, the semantics of dLSC corresponds directly to that of LSC whenever violations of charts coincide with the charts being enabled at the same location of the run.

The semantics of dLSC somewhat resembles that of the existential, conditional, interpretation of LSC in (Sibay et al., 2008), which demands that whenever a run ends with a prechart of an LSC, there exists a run that continues with the main chart. In dLSC, however, all possibilities to continue are induced by the specification, and progress is assumed whenever there is an enabled main chart.\(^1\) Moreover, again, dLSC employs composition of partial orders whereas (Sibay et al., 2008) interprets LSCs over sequential runs.

Distributed LSCs are closely related to oclets (Fahland, 2009; Fahland, 2010), a scenario-based formalism which employs the prechart/main chart distinction of LSCs in terms of Petri nets and their partially ordered runs. dLSC can be seen as a reduction of the idea of oclets to a purely event-based formalism in the context of LSCs. This adaptation significantly simplifies the formalism, and yet dLSCs essentially subsume oclets, as the latter can be expressed in terms of dLSCs. Moreover, in dLSC, as in LSC, a main chart is completely synchronized after the prechart. This allows eliminating oclets’ notion of implied scenarios (Uchitel et al., 2001), i.e., additional behavior that is not explicitly specified. The framework of oclets also allows to extract decentralized components (Fahland, 2010) from a specification, but it requires a different method.

\(^1\)One can slightly generalize the definition of dLSC to also allow for charts for which, when enabled, progress is not assumed. The synthesis algorithm, with trivial changes, would still apply.
that cannot take all specifications as input, and has exponential worst-case complexity unlike the complete, polynomial, method proposed in this paper.

Synthesis of systems and decentralized components from scenario-based specifications is a well-known problem, with many contributions; (Liang et al., 2006) provides an extensive survey. Most approaches consider (H)MSCs or UML Sequence Diagrams as input, and translate the specification into model structures of Statecharts or Petri nets somewhat similarly to our technique (Liang et al., 2006), or through behavioral synthesis (Uchitel et al., 2001) in the synthesized system (Bergenthum et al., 2009). Moreover, centralized synthesis from LSC has been studied, which succeeds by structural translations to Statecharts (Harel and Kugler, 2002), or through game-based synthesis techniques (Harel and Segall, 2012). However, in all cases, synthesis introduces non-specified behavior (i.e., implied scenarios (Liang et al., 2006)), or non-specified synchronization among events, which makes the extraction of decentralized components impossible. Our synthesis technique makes synchronization information part of the exchanged messages by means of token histories, which effectively prohibits implied scenarios, and, for a large class of specifications, limits the synchronization among components to the interactions specified in the dLSCs.

8 CONCLUSION

In this paper, we take a fresh look at an essential fragment of live sequence charts (LSC) and provide a semantics in terms of partially ordered runs by means of simple scenario composition. This variant of LSC, called distributed LSC (dLSC), has sufficient expressive power for specifying concurrent systems: the class of systems that can be specified with dLSC strictly contains the class of systems that can be modeled with classical Petri nets. We also provide a technique to synthesize, from any dLSC specification, an implementation in the class of token history nets (THPN). Decentralized components can easily be extracted from the THPN. The approach has polynomial time and space complexity and is implemented in a tool.

Our work allows for much future work. One may extend dLSC with a notion of data, such as from algebraic specifications; we believe our synthesis technique still applies, as coloured Petri nets, which support data manipulation, could be similarly synthesized. Additionally, operations on dLSC specifications, such as scenario (de-)composition and refinement, are of interest. They would permit to systematically develop and reason on complex dLSC specifications.

Acknowledgement. We thank David Harel for his helpful comments on this work. The research was supported in part by the John von Neumann Minerva Center for the Development of Reactive Systems at the Weizmann Institute of Science, and by an Advanced Research Grant to David Harel from the European Research Council (ERC) under the European Community’s FP7 Programme.

REFERENCES


Liang, H., Dingel, J., and Diskin, Z. (2006). A Comparative Survey of Scenario-Based to State-Based Model Syn-
thesis Approaches. In SCESM ’06, pages 5–12, New York, NY, USA. ACM.

APPENDIX

A.1 Distributed LSC Formalized

In the following we assume a set Σ of actions. It includes names (or, labels) for all the events that one wishes to refer to in a specification.

**Labeled Partial Orders.** We begin with some preliminary definitions. A labeled partial order (lpo) is a tuple \( l = (E, \prec, \lambda) \), where \( E \) is (a possibly infinite) set of events, \( \prec \subseteq (E \times E) \) is a strict partial order relation on \( E \) (i.e., irreflexive and transitive)\(^2\), \( \lambda : E \to \Sigma \) is a labeling function, and, furthermore, for any event \( x \in E \), the set \( \{ y \in E : y \leq x \} \) is finite. The last property is included to restrict the form of lpo’s so that they correspond to realizable traces of executions.

Let \( l = (E, \prec, \lambda) \) be an lpo. For any \( A \subseteq E \), the restriction of \( l \) to \( A \) is defined by \( l|_A = (A, \prec \cap (A \times A), \lambda|_A) \), which is again an lpo. Let \( l' = (E', \prec', \lambda') \) be another lpo. \( \varphi : E \to E' \) is an isomorphism from \( l \) onto \( l' \), denoted \( l \cong l' \), if \( \varphi \) is a one-to-one function from \( E \) onto \( E' \), for any events \( x, y \in E \) holds \( x < y \) iff \( \varphi(x) < \varphi(y) \), and \( \lambda' \circ \varphi = \lambda \). \( l \) is isomorphic to \( l' \), denoted \( l \cong l' \), if there is \( \varphi \) such that \( l \cong \varphi l' \). \( l \) is said to be finite (resp., empty) if \( E \) is finite (resp., empty).

We write max(\( l \)) (resp., min(\( l \))) for the maximal (resp., minimal) events in \( E \). Given a finite \( A \subseteq E \), we say that \( A \) is a maximal dense set in \( l \) if \( \max(l|_A) \subseteq \max(l) \), and for any \( x, y \in A \) and \( z \in E \), if \( x < z < y \) then \( z \in A \). Given such \( A \) and another lpo \( l'' = (E'', \prec'', \lambda'') \) such that \( E \cap E'' = \emptyset \), the local concatenation of \( l'' \) after \( A \) in \( l \), is defined by \( l[A] \to l'' = (E \cup E'', \prec \cup \prec'', \{ (x \in E : \exists y \in A : x < y \} \times E'') \), \( \lambda \cup \lambda'' \). It is easy to verify that it is again an lpo. As a special case, the (global) concatenation of \( l \) after \( l' \) is defined by \( l \to l' = (l[E] \to l') \).

**Abstract Syntax of Distributed LSC.** We are now ready to present the abstract syntax of distributed LSCs. A distributed LSC (dLSC) is a tuple \( l = (l_p, l_m) \), where \( l_p \) and \( l_m \) are finite nonempty lpo’s. \( l_p \) is called the prechart of \( L \), and \( l_m \) is called the main chart of \( L \). As we assume complete synchronization between the main chart and the prechart, we technically separate the chart into two lpo’s. A dLSC specification consists of a tuple \( S = \langle D, R_0 \rangle \), where \( D \) is a finite set of dLSCs, and \( R_0 \) is a finite lpo called the initial run.

**Semantics of Distributed LSC.** We hereby turn to the semantics of dLSC specifications. Given an lpo \( l = (E, \prec, \lambda) \) and a dLSC \( L = \langle l_p, l_m \rangle \), we first define the set of all continuations of \( l \) according to \( L \), which is denoted by \( \triangleright L \). Put \( l_m = (E_m, \lambda_m) \), and, without loss of generality, assume that \( E \cap E_m = \emptyset \) (we may always take \( l'_m \cong l_m \) satisfying this constraint). Then, \( \triangleright_L \) is the set of lpo’s of the form \( l[A] \to l_m \), where \( A \subseteq E \) is a finite maximal dense set in \( l \) such that \( l[A] \cong l_p \). Moreover, given a set \( D \) of dLSCs, the set of all continuations of \( l \) according to the charts in \( D \) is defined by \( \triangleright_D = \bigcup_{L \in D} \triangleright_L \).

Given a dLSC specification \( S = \langle D, R_0 \rangle \), a construction sequence for \( S \) is a sequence of lpo’s \( \rho \) with domain \( 0 < D \leq \mathbb{N} \) (i.e., \( D \) is either the set \( \mathbb{N} \) of all natural numbers, or a natural number \( n > 0 \), in the sense that \( D = n = \{ i \in \mathbb{N} : i < n \} \)), satisfying the following: \( \rho_0 = R_0 \), and for all \( i \in D \) such that \( i + 1 \in D \) holds \( \rho_{i+1} \cong \rho_i \triangleright_D \). For each \( i \in D \), put \( \rho_i = (E_i, \prec_i, \lambda_i) \). Then, the value of \( \rho \) is defined to be the lpo corresponding to the limit of the sequence, which can be formally defined by

\[
\val(\rho) = \left\{ \bigcup_{i \in D} E_i : \bigcup_{i \in D} \prec_i, \bigcup_{i \in D} \lambda_i \right\}.
\]

It is easy to verify that \( \val(\rho) \) is, in turn, a (possibly infinite) lpo, and that if \( D \) is finite then \( \val(\rho) \).
is the last lpo in the sequence. The construction sequence \( \rho \) is said to be final if also \( \text{val}(\rho) \cap \mathcal{D} = \emptyset \). The (denotational) semantics of a dLSC specification \( S \) is an lpo language defined by \( L(S) = \{ \text{val}(\rho) : \rho \text{ is a final construction sequence for } S \} \).

### A.2 Token History Nets

**Syntax.** A token history (over an alphabet \( \Sigma \)) is a finite lpo \( l = \langle E, <, \lambda \rangle \) with labeling \( \lambda : E \rightarrow \Sigma \cup \{ \tau \} \). \( \Sigma \) and \( \tau \) will include the labels of transitions; \( \Sigma \) are observable actions (which will represent the actions in a dLSC specification), while \( \tau \) is a silent (or, unobservable) action. Let \( L \) denote the set of all token histories over \( \Sigma \).

A THPN \( N = (P, T, F, m_0, \lambda, g, w) \) consists of places \( P \), transitions \( T \) (where \( P \cap T = \emptyset \)), and arcs \( F \subseteq (P \times T) \cup (T \times P) \). The initial marking \( m_0 \) assigns each place a finite multiset \( m_0(p) \in \mathbb{N}^\Sigma \) of lpo’s (token histories). The labeling function \( \lambda : T \rightarrow \Sigma \cup \{ \tau \} \) assigns each transition a label. The function \( g \) assigns each transition a guard; here we are only interested in a uniform guard that is uniquely determined by an lpo, thus \( g : T \rightarrow L \). We consider THPNs with weighted arcs, in which \( w : F \rightarrow \mathbb{N} \setminus \{0\} \) assigns positive weights to arcs. Let \( *t = \{ p \in P : (p, t) \in F \} \) and \( t^* = \{ p \in P : (t, p) \in F \} \) denote the pre-places and the post-places of transition \( t \), respectively.

A token history net is depicted in Fig. 7a. As usual, a circle represents a place, a rectangle represents a transition, and transition labels are inscribed. Moreover, each transition has a guarding lpo (shown for the transitions that go by the names \( L_2 \) to \( L_5 \), and \( L_7 \)). In the net of Fig. 7a, all arcs have weight 1.

**Semantics.** The semantics of THPN slightly extends classical Petri net semantics. When a transition \( t \) fires, it consumes tokens (i.e., token histories) from its pre-places, merges them into a single lpo, appends a new event labeled \( \lambda(t) \), and produces the resulting token history on each post-place of \( t \). An lpo is enabled if and only if the consumable token histories satisfy the guard \( g(t) \); i.e., the resulting token history ends with \( g(t) \). The technical details are as follows.

1. **For two lpo’s \( l_1, l_2 \in L \), their union is defined by \( l_1 \cup l_2 := \langle E_1 \cup E_2, \{<_{1} \cup <_2\}^+, \lambda_1 \cup \lambda_2 \rangle \). In the execution of a THPN, this operation is well defined as the lpo’s are consistent (see van Hee et al., 2008)). For a finite multiset of lpo’s \( S \in \{ l_1, \ldots, l_n \} \), let \( \cup S := l_1 \cup \ldots \cup l_n \).
2. **For an lpo \( l = \langle E, <, \lambda \rangle \in L \) and an action \( a \in \Sigma \), the concatenation of \( l \) with \( a \) is defined by \( l \rightarrow a := \langle E \cup \{e\}, < \cup \{\langle e', e \rangle : e' \in E \}, \lambda \cup \{(e, a)\} \rangle \) where \( e \not\in E \) is assumed to be a globally fresh event (never used before).
3. **A (firing) mode of a transition \( t \) is an assignment \( \beta : *t \rightarrow \mathbb{N}^\Sigma \), specifying for each preplace \( p \) of \( t \), a multiset of token histories \( \beta(p) \) s.t. \( |\beta(p)| = w(p, t) \), which are to be consumed. Let \( S := \bigcup_{p \in *t} \beta(p) \) be their (multiset) union. The expression \( f(t, \beta) := \bigcup S \rightarrow \lambda(t) \) describes the token history obtained when \( t \) fires in mode \( \beta \); it is the union of all consumed token histories, extended by the action \( \lambda(t) \).
4. **For two lpo’s \( l_1, l_2 \in L \), we say that \( l_1 \) ends with \( l_2 \), written \( l_1 \prec l_2 \), if \( l_1 \) has a maximal dense set \( A \) in \( l_2 \) (see Sect. A.1) s.t. \( l_1|A \equiv l_2 \).
5. **For an lpo \( l = \langle E, <, \lambda \rangle \in L \), the restriction of \( l \) to \( \Sigma \), written \( l|_\Sigma \), is the restriction \( \{ l|_{S} \} \) of \( l \) to all observable events \( E_{\Sigma} = \{ e \in E : \lambda(e) \in \Sigma \} \).
6. **Let \( m \) be a marking of \( N \). Transition \( t \) is enabled at \( m \) in mode \( \beta \) if for each \( p \in *t \), \( \beta(p) \subseteq m(p) \) (as multisets), and \( f(t, \beta) \prec m(t) \prec g(t) \); that is, a firing of \( t \) would produce a token history that ends with the guarding lpo of \( t \) (ignoring silent events in the token history).
7. **If \( t \) is enabled at \( m \) in \( \beta \), then \( t \) can fire, which results in a new marking \( m' \). For each place \( p \in P \), first let \( m''(p) := m(p) \setminus \beta(p) \) if \( p \in *t \), and \( m''(p) := m(p) \) otherwise. Then, \( m'(p) := m''(p) \cup w(t, p) \gamma(f(t, \beta)) \) if \( p \in *t \) (where \( w(t, p) \gamma(f(t, \beta)) \) denotes the multiset with \( w(t, p) \) instances of \( f(t, \beta) \)), and \( m'(p) := m''(p) \) otherwise. Here, \( \cup \) and \( \setminus \) denote multiset union and subtraction as usual.

### A.3 Synthesizing Token History Petri Nets from dLSC Specifications

**Translating the specification.** We translate a dLSC specification \( S = \langle D, R_0 \rangle \) over actions \( \Sigma \) (where \( D \) is a set of dLSCs and \( R_0 \) is the initial run) into an equivalent THPN \( N_S \) over \( \Sigma \). We first translate each chart \( L \in D \) into a net \( N_L \), and then, compose the resulting nets to form the net \( N_S \) of the entire specification. The different \( N_L \)'s are connected via shared places: each maximal main-chart event labeled \( a \) of some chart \( L \in D \) defines a shared place \( p_a \), on which \( N_L \) produces. For a chart \( L' \in D \), in which an event labeled \( a \) appears maximal in the prechart, \( N_L \) will consume from \( p_a \). We first define the shared places, and then show how to translate a dLSC \( L \in D \) into \( N_L \).

**Shared Places.** Let \( \Sigma_{\text{max}} \subseteq \Sigma \) denote the names (i.e., labels) of all events that are maximal in the main
Auxiliary notions on dLSCs. Let \( L = \langle l_p, l_m \rangle \) be a dLSC. Technically, let \( l_L := l_p \rightarrow l_m \) denote the lpo where \( l_m \) is appended to \( l_p \). Given a main-chart event \( e \in E_m \), the events that precede it are \( E_e := \{ e' \in E_L : e' \leq L e \} \). The local history of \( e \) in \( L \) is the restriction of \( l_L \) to \( E_e \), written \( h_L(e) := l_L|_{E_e} \). Moreover, we write \( f \leq L e \) if \( f \leq_L e \) and for no event \( g \in E_L \), holds \( f \leq_L g \leq_L e \). Let \( \text{pre}_L(e) := \{ f \in E_L : f \leq_L e \} \), and \( \text{post}_L(e) := \{ f \in E_L : e \leq_L f \} \) denote the set of direct predecessor events and direct successor events of \( e \) in \( L \), respectively.

Translating individual charts. Each dLSC \( L = \langle l_p, l_m \rangle \) in the specification induces a net \( N_L \). For each main-chart event \( e \in E_m \) a transition \( t_e \) is defined in \( N_L \), labeled \( \lambda(t_e) := \lambda(e) \). The partial order of \( t_m \) is encoded by places.

- For any \( e, f \in E_m \) with \( e \leq f \) (direct order), define a place \( p_{e, f, L} \) “between” \( t_e \) and \( t_f \), and arcs \( \langle t_e, p_{e, f, L}, p_{e, f, L}, t_f \rangle \in F \).
- For any \( e \in \text{min}(l_m) \) define an activation place \( p_{0, e, L} \) and an arc \( \langle p_{0, e, L}, t_e \rangle \in F \).
- For any \( e \in \text{max}(l_m) \) define arc \( \langle t_e, p_{\lambda_m(e)}, F \rangle \in F \), into the matching shared place.

All arcs have weight 1. Moreover, transition \( t_e \) has the local history of \( e \) as the guarding lpo; i.e., \( g(t_e) := h_L(e) \). This guard ensures that \( t_e \) is only enabled if the tokens produced by \( e \) will indeed exactly end with the entire local history \( h_L(e) \) of \( e \). The initial run \( R_0 \) is translated into a net \( N_{R_0} \) in the same way (treated as a main chart of an ordinary dLSC).

The transitions of the minimal events in the main chart of \( L \) shall only be enabled when all the maximal events in \( L \)’s prechart have occurred. We formalize this by a main-chart activation transition \( t_L \) with label \( \lambda(t_L) := \tau \) (unobservable).

- \( t_L \) consumes from the shared places of the maximal events in \( L \)’s prechart: \( \langle p_a, t_L \rangle \in F \) for each \( a \) that is a label of an event in \( \text{max}(l_p) \), with arc weight \( w(\langle p_a, t_L \rangle) = | \{ e \in \text{max}(l_p) : \lambda(p_a(e) = a) \} | \); and
- \( t_L \) produces on the activation places of the minimal events in \( L \)’s main chart: \( \langle t_L, p_{0, e, L} \rangle \in F \) for each \( e \in \text{min}(l_m) \), with arc weight 1.

Transition \( t_L \) has the prechart \( l_p \) as guard \( g(t_L) := l_p \). Thus, \( t_L \) only synchronizes tokens that together have seen the entire prechart of \( L \). A firing of \( t_L \) will not be visible in the resulting token history as \( t_L \) has a label \( \tau \). E.g., in Fig. 7a, the \( \tau \) transition \( L_2 \) is the activation transition of dLSC \( L_2 \) of Fig. 2b.

Finally, we obtain the complete net \( N_S \), for the entire specification \( S = \langle D, R_0 \rangle \), by constructing the component-wise union \( N_S := N_{R_0} \cup \bigcup_{L \in C} N_L \) (assuming that the nets are pairwise disjoint except for the shared places \( P_{\text{shared}} \)). The initial marking \( m_0 \) of \( N_S \) contains an empty token history in each activation place of \( N_{R_0} \); i.e., for each \( e \in \text{min}(R_0) \), \( m_0(p_{0, e, R_0}) := \{(0,0,0)\} \), while other places are empty. Note that the entire synthesis has polynomial time and space complexity in the given specification.

**A.4 Extracting Components from the Synthesized Net**

Let \( N_S = \langle P, T, F, m_0, \lambda, g, w \rangle \) be the THPN synthesized from a dLSC specification \( S \) over a finite set \( C \) of components. Additionally, let \( T = \{ T_i \}_{i \in C} \) be a partitioning of the transitions of \( N_S \) into components; it is obtained according to the action labels of the transitions and by further assigning the \( \tau \)-labeled activation transitions to components. The component of \( N_S \) induced by \( T_i \) is the subnet of \( N_S \) consisting of transitions \( T_i \) and their pre- and post-places \( P_i = \bigcup_{t \in T_i} \bullet t \cup \{ \bullet \} \); i.e., the net \( N_i = \langle P_i, T_i, F_i, m_0_i, T_i \rangle \), where \( F_i := F|_{T_i} \cup P_i \).