Tussentoets Wiskunde 2 (2DD50)
Donderdag 28 November 2013, 10:00–11:00

This exam consists of 2 problems on 2 pages.
The maximum score is 15 points.
Provide your answers in Dutch or in English.
Provide a concise and clear motivation for all yours answers.

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Problem 1. TU Eindhoven maintains a powerful mainframe computer for research use. During all working hours, an operator must be able to operate and maintain the computer, as well as to perform programming services. At the beginning of the semester, there arises the problem of assigning different working hours to the operators. Because all the operators are currently enrolled in the university, they are available to work only a limited number of hours each day, as shown in the following table.

		Maximum hours of availability				
Operator	Wage rate	Mon	Tue	Wed	Thu	Fri
Alice	€31/hour	3	0	0	6	2
Bob	€26/hour	0	6	0	6	0
Carol	€24/hour	4	8	4	0	4
Dana	€23/hour	6	0	6	0	6
Edgar	€30/hour	3	0	3	8	0
Flora	€25/hour	5	5	5	0	5

There are six operators: the four undergraduate students Alice, Bob, Carol and Dana, and the two graduate students Edgar and Flora. They all have different wage rates because of differences in their experience with computers and in their programming abilities. The above table shows their wage rates, along with the maximum number of hours that each can work each day. Each operator is guaranteed a certain minimum number of hours per week: the undergraduate students are each guaranteed 8 hours per week, and the graduate students are each guaranteed 7 hours per week.

The computer facility is open for operation from 8:00 to 22:00, Monday through Friday, with **exactly one** operator on duty during these hours. Because of a tight budget, TU Eindhoven has to minimize cost and optimizes working hours down to the last split second.

(a) [4 points] Formulate a linear programming model for this problem. State the variables, the objective function, and the constraints.

(b) [2 points] Explain the meaning of your variables and constraints, and justify your objective function.

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Problem 2. Consider the following linear program:

$$
\begin{array}{rrrrrcr}
\min Z & = & x_1 & +2x_2 & +3x_3 & & \\
\text{s.t.} & & x_1 & +2x_2 & -3x_3 & \leq & 6 \\
 & & -2x_1 & +3x_2 & -x_3 & = & 3 \\
 & & -3x_1 & +2x_2 & +x_3 & \leq & -4 \\
\text{with} & & \multicolumn{5}{l}{x_1 \text{ free}, \; x_2 \geq 0, \; x_3 \leq 0}
\end{array}
\qquad (LP)
$$

(Attention: Note that variable x_3 has to be **non-positive**.)

(a) [2 points] Bring the linear program (LP) into our standard form with equality constraints. (Turn it into a maximization problem, introduce appropriate slack- and surplus variables, etc.)

(b) [4 points] Solve the linear program by applying the **2-phase method**. Show every intermediate tableau, and clearly indicate your pivot rows and pivot columns.

(c) [1 point] Does the linear program (LP) have a unique optimal solution? Is it unbounded? Is it infeasible?

(d) [1 point] Formulate the dual of the linear program (LP).

(e) [1 point] Does the dual have a unique optimal solution? Is it unbounded? Is it infeasible?