

This exam consists of 2 problems on 2 pages.
The maximum score is 15 points.
Provide your answers in Dutch or in English.
Provide a concise and clear motivation for all your answers.

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Problem 1. The company SPYKER produces luxury cars. The company management is currently fixing the production schedule for the forthcoming year, which essentially means to decide how many cars should be produced in each of the coming four quarters. The following table lists the demands that are to be satisfied at the end of each quarter:

	Demand
Quarter 1	40
Quarter 2	60
Quarter 3	75
Quarter 4	26

The decision making process depends on the following production and inventory costs.

- During each quarter, SPYKER can produce up to 40 cars with its regular employees. The regular production costs per car amount to €80.000.
- By hiring additional workers, SPYKER can produce additional cars. The production costs per such additional car amount to €100.000.
- At the beginning of the first quarter, SPYKER already has 10 cars on stock.
- Every car that remains on stock at the end of a quarter causes inventory costs of €8.000.

The goal is to find a production plan that minimizes the overall cost while satisfying all demands.

- [4 points] Formulate a linear programming model for this problem. State your variables, the objective function, and the constraints.
- [2 points] Explain the meaning of your variables and constraints, and justify your objective function.

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Problem 2. Consider the following linear program:

$$\begin{aligned} \max Z &= 5x_1 - x_2 \\ \text{s.t.} \quad &x_1 + x_2 \leq -1 \\ &x_1 - x_2 \geq 3 \\ &-2x_1 + x_2 \geq -2 \\ \text{with} \quad &x_1 \geq 0; \quad x_2 \leq 0 \end{aligned} \tag{LP.1}$$

- (a) [2 points] Bring the linear program (LP.1) into our standard form with equality constraints. (Introduce appropriate slack- and surplus variables, ensure non-negativity constraints and non-negative right hand sides, etc.)
- (b) [4 points] Solve the linear program by applying the **Big-M method**. Show every intermediate tableau, and clearly indicate your pivot rows and pivot columns.
- (c) [2 points] Formulate the dual (LP.2) of the linear program (LP.1).
- (d) [1 point] Does the dual (LP.2) have a finite optimal solution? Is it unbounded? Is it infeasible?

Remember our grading rules:

- We only grade up to the **first mistake** you make on a problem
- **Zero points** for anything you write **after** the first mistake
- **Zero points** for writing wrong solutions (even if you write a lot)
- **Zero points** for solving questions that we did not ask
- In particular: **No points** for solving 2(b) by the 2-phase method, or for guessing the answer
- In particular: **No points** for dualizing a modification of (LP.1) under 2(c)
- **Zero points** if you submit two different solutions for one problem