Tussentoets Wiskunde 2 (2DD52)
Donderdag 27 November 2014, 11:00–12:30

This exam consists of 2 problems on 2 pages.
The maximum score is 15 points.
Provide your answers in Dutch or in English.
Provide a concise and clear motivation for all yours answers.

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Problem 1. The Soda-Drink company APS owns four natural water springs, and holds a production plant at each of these springs. APS produces three Soda types: APS blue, APS green, and APS red. The APS head quarters determine the amounts of Soda that each plant should produce in the forthcoming period. Relevant factors are the number of available cubic meters of water (note that $1m^3$ of water yields $1m^3$ of Soda), and the number of available working hours per plant as indicated in the following table:

Plant location	Maximal number of available m^3	Available working hours
Gent	400	1800
Liege	900	3200
Mons	550	2300
Namur	350	1200

Each of the three Soda types has a certain (expected) profit and a certain number of needed working hours. Furthermore, for each Soda type there is an upper limit on the number of produced cubic meters, the so-called production ceiling:

Soda type	Production ceiling (in m^3)	Needed working hours per m^3	expected profit per m^3 in €
Blue	700	2	550
Green	800	4	400
Red	300	3	350

The goal of APS head quarters is of course to maximize the overall expected profit.

(a) [4 points] Formulate a linear programming model for this problem. State your variables, the objective function, and the constraints.

(b) [2 points] Explain the meaning of your variables and constraints, and justify your objective function.

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Problem 2. Consider the following linear program:

$$
\begin{array}{llrrrcr}
\min Z & = & -3x_1 & +x_2 & +x_3 & & \\
\text{s.t.} & & x_1 & +x_2 & +x_3 & \leq & -1 \\
 & & -x_1 & +x_2 & & \leq & -3 \\
 & & -2x_1 & -x_2 & -x_3 & \geq & -2 \\
\text{with} & & \multicolumn{5}{l}{x_1 \geq 0; \quad x_2, x_3 \leq 0}
\end{array}
\qquad \text{(LP.1)}
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(a) [2 points] Bring the linear program (LP.1) into our standard form with equality constraints. (Introduce appropriate slack- and surplus variables, take care of non-negativity constraints and non-negative right hand sides, etc.)

(b) [4 points] Solve the linear program by applying the **2-phase method**. Show every intermediate tableau, and clearly indicate your pivot rows and pivot columns.

(c) [2 points] Formulate the dual (LP.2) of the linear program (LP.1).

(d) [1 point] Does the dual (LP.2) have a unique optimal solution? Is it unbounded? Is it infeasible?