Hints to some recommended exercises

(TU/e, 2DD50, Fall 2015)

All exercise numbers refer to the **10th edition** of "Introduction to Operations Research" by F.S. Hillier and G.J. Lieberman.

3.1-2 Draw the bounding lines.

3.2-3(a,b) As in the Wyndor Glass problem, we want to find the optimal levels of two activities that compete for limited resources. We want to find the optimal mix of the two activities. Let x_1 be the fraction purchased in the first venture; let x_2 be the fraction purchased in the second venture.

$$\begin{aligned} \max Z = \quad & 4500x_1 + 4500x_2 \\ \text{subject to} \quad & x_1 \leq 1 \\ & x_2 \leq 1 \\ & 5000x_1 + 4000x_2 \leq 6000 \\ & 400x_1 + 500x_2 \leq 600 \\ & x_1, x_2 \geq 0 \end{aligned}$$

3.3-2 Proportionality and divisibility: yes; Additivity: no; Certainty: perhaps.

3.4-12(a,b) maximize $1.4A_4 + 1.7B_3 + 1.9C_2 + 1.3D_5 + R_5$

subject to the constraints

$$\begin{aligned} & A_1 + B_1 + R_1 = 60000 \\ & A_2 + B_2 + C_2 + R_2 = R_1 \\ & A_3 + B_3 + R_3 = R_2 + 1.4A_1 \\ & A_4 + R_4 = r_3 + 1.4A_2 + 1.7B_1 \\ & D_5 + R_5 = R_4 + 1.4A_3 + 1.7B_2 \\ & A_t, B_t, C_t, D_t, R_t \geq 0 \end{aligned}$$

3.4-15(a)

- Number the operators K.C, D.H, H.B, S.C, K.S, N.K by 1,2,3,4,5,6.
- Number the five days by 1,2,3,4,5.
- For $1 \leq i \leq 6$, let x_i denote the working hours of the ith operator per week
- For $1 \leq i \leq 6$ and $1 \leq j \leq 5$, let $y_{i,j}$ denote the working hours of the ith operator on the jth day.
- For $1 \leq i \leq 6$ and $1 \leq j \leq 5$, let $a_{i,j}$ denote the number of availability hours of the ith operator on the jth day (as specified in the table)

- Objective function: minimize $25x_1 + 26x_2 + 24x_3 + 23x_4 + 28x_5 + 30x_6$.
- Connection between x-variables and y-variables for $1 \leq i \leq 6$:
  $y_{i,1} + y_{i,2} + y_{i,3} + y_{i,4} + y_{i,5} = x_i$
- Maximum availability hours per day for $1 \leq i \leq 6$ and $1 \leq j \leq 5$:
  $y_{i,j} \leq a_{i,j}$
- Guaranteed minimum working hours:
  $x_1, x_2, x_3, x_4 \geq 8$ and $x_5, x_6 \geq 7$
- Covering 14 working hours per day, for $1 \leq j \leq 5$:
  $y_{1,j} + y_{2,j} + y_{3,j} + y_{4,j} + y_{5,j} + y_{6,j} = 14$
- Non-negativity constraints for $1 \leq i \leq 6$ and $1 \leq j \leq 5$:
  $x_i \geq 0$ and $y_{i,j} \geq 0$

4.6-3(a) maximize $-2x_1 - 3x_2 - x_3$

subject to the constraints

$$x_1 + 4x_2 + 2x_3 - s_1 = 8$$
$$3x_1 + 2x_2 - s_2 = 6$$
$$x_1, x_2, x_3, s_1, s_2 \geq 0$$

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4.4-3(a,c) For (a), draw a picture. The CPF solutions are $(0,0)$, $(0,40)$, $(25,0)$, $(20,20)$.

For (e), the answer is $x_1 = 20$, $x_2 = 20$, $Z = 60$.

4.4-7 $x_1 = 3/2$, $x_2 = 1/2$, $x_3 = 0$, $Z = 5/2$.

4.4-8 $x_1 = 20/3$, $x_2 = 0$, $x_3 = 110/3$, $Z = 200/3$.

4.5-4 unbounded.

4.5-8 $Z = 5$ with four optimal BF solutions:

(1)	$x_1 = 0$	$x_2 = 3$	$x_3 = 0$	$x_4 = 2$
(2)	$x_1 = 0$	$x_2 = 3$	$x_3 = 2$	$x_4 = 0$
(3)	$x_1 = 3$	$x_2 = 0$	$x_3 = 0$	$x_4 = 2$
(4)	$x_1 = 3$	$x_2 = 0$	$x_3 = 2$	$x_4 = 0$

5.1-6 Draw the bounding lines.

5.1-9 (a) false; (b) false; (c) false

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4.6-6 infeasible

4.6-7(a,b,c) $x_1 = 0$, $x_2 = 0$, $x_3 = 50$, $Z = 150$

4.6-9(a,b) $x_1 = 0$, $x_2 = 15$, $x_3 = 15$, $Z = 90$

4.6-16(a,b,c) unbounded; for instance choose $(x_1, x_2, x_3, x_4) = (0, 0, -2t, t+1)$ and let t go to infinity.

5.1-4 (a) $(x_1, x_2, x_3) = (10, 0, 0)$. (b) $x_2 = 0$ and $x_3 = 0$ and $x_1 - x_2 + 2x_3 = 10$.

5.1-10 (a) true; (b) true; (c) false

5.1-11 (a) false; (b) false; (c) true

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5.3-1 (a) The final tableau is:

	Z	x_1	x_2	x_3	x_4	x_5	x_6	
Z	1	2	0	0	1	1	0	8
x_2	0	5	1	0	1	3	0	14
x_6	0	2	0	0	0	1	1	5
x_3	0	4	0	1	1	2	0	11

(b) $2x_1 - 2x_2 + 3x_3 = 5$, and $x_1 + x_2 - x_3 = 3$, and $x_1 = 0$

5.3-2 (a) The final tableau is:

	Z	x_1	x_2	x_3	x_4	x_5	x_6	
Z	1	3	0	2	0	1	1	9
x_2	0	1	1	-1	0	1	-1	1
x_4	0	2	0	3	1	-1	2	3

(b) $4x_1 + 2x_2 + x_3 + x_4 = 5$, and $3x_1 + x_2 + 2x_3 + x_4 = 4$, and $x_1 = 0$, and $x_3 = 0$

6.1-3 (a) The dual has fewer constraints. (b) The primal has fewer constraints.

6.1-4 (a) The dual is:

$$\begin{aligned} \min W = \quad & 12y_1 + y_2 \\ \text{subject to} \quad & y_1 + y_2 \geq -1 \\ & y_1 + y_2 \geq -2 \\ & 2y_1 - y_2 \geq -1 \\ & y_1, y_2 \geq 0 \end{aligned}$$

(b) $(y_1, y_2) = (0, 0)$ is feasible for the dual.

6.1-5(a,b) (a) The dual is:

$$\begin{aligned} \min W = \quad & 3y_1 + 5y_2 \\ \text{subject to} \quad & y_1 \geq 2 \\ & y_2 \geq 6 \\ & y_1 + 2y_2 \geq 9 \\ & y_1, y_2 \geq 0 \end{aligned}$$

(b) The optimal solution for the dual is $(y_1, y_2) = (2, 6)$

6.1-7 (There is a typo in the problem statement. The right hand side of the second constraint should read "-10" instead of the current "10".) Then the dual is:

$$\begin{aligned} \min W = \quad & 20y_1 - 10y_2 \\ \text{subject to} \quad & 4y_1 - y_2 \geq 2 \\ & y_1 + y_2 \geq 3 \\ & y_1, y_2 \geq 0 \end{aligned}$$

If we choose $(y_1, y_2) = (t, 3t)$ and let t go to infinity, the (dual) objective value becomes arbitrarily small.

6.4-2 Read the slides for College 3.

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9.1-2 $32,400$ miles

9.1-4 (a) Introduce a dummy product with demand 400. The table then looks as follows:

	700	1000	900	400
400	41	55	48	0
600	39	51	45	0
400	42	56	50	0
600	38	52	–	0
1000	39	53	–	0

(b) $\$121,200$

9.2-4(a-c,e) There are 7 variables in every BF, and 3 of them are degenerate.

The optimal cost is 13.

9.2-9(a-c) As the total energy demand is 60 units, the maximal needed supply of gas and electricity is respectively 60 units. Introduce a dummy demand of 140 units to balance supply and demand.

		Electricity	H-Water	H-Space	Dummy
		20	10	30	90
Electricity	60	50	90	80	0
Natural Gas	60	–	60	50	0
Solar	30	–	30	40	0

The optimal cost is \$2600.

9.3-1(a,b) The optimal cost is 20.

9.3-2 The optimal cost is \$2100.

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11.2-2(b) The optimal value is 140.

11.3-2 The optimal value is 25.

11.3-3 The optimal value is 23.