

# Discrete Structures 2IT50

Interim test September 23, 2015, 13:45 - 14:45

This interim test is the first of three, of which the best two count for 30% of the final grade.

In giving proofs you may use theorems and lemmas from the lecture notes (not exercises), as long as you indicate that you use them.

The test consists of three problems each having the indicated weight.

Please indicate in which of the following instruction groups you are:

- Group 01, AUD 12, Astrid Pieterse,
- Group 02, LaPlace 1.105, Tom Verhoeff,
- Group 03: Metaforum 7, Wieger Wesselink,
- Group 04: Metaforum 8, Jaap van der Woude.

## Problem 1.

(20 %) The equivalence relation  $R$  on  $\{1, 2, 3, 4, 5\}$  is given by

$$R = \{(x, y) \mid (x - 2)^2 = (y - 2)^2\}.$$

Determine the equivalence classes of  $R$ , motivate your answer.

## Problem 2.

Let  $R, S$  be relations on a set  $U$  satisfying  $R; S \subseteq R$ .

1. (20 %) Prove that  $(S; R)^n \subseteq S; (R^n)$  for all  $n \geq 1$ .
2. (20 %) Prove that  $(S; R)^+ \subseteq S; (R^+)$ .

## Problem 3.

Let  $(V, E)$  be a connected undirected graph in which every node has odd degree.

1. (20 %) Prove that  $\#V$  is even.
2. (20 %) Prove that the undirected graph  $(V \times \{1, 2\}, E')$  with

$$E' = \{(v, 1), (v, 2) \mid v \in V\} \cup \{(v, i), (w, i) \mid (v, w) \in E \wedge i \in \{1, 2\}\}$$

admits an Euler cycle.