

Discrete Structures 2IT50

Interim test October 7, 2015

This interim test is the second of three, of which the best two count for 30% of the final grade.

In giving proofs you may use theorems and lemmas from the lecture notes (not exercises), as long as you indicate that you use them.

The problems may be solved either in English or in Dutch.

This test will be graded according to the given percentages.

Please indicate in which of the following instruction groups you are:

- Group 01, AUD 12, Astrid Pieterse,
- Group 02, LaPlace 1.105, Tom Verhoeff,
- Group 03: Metaforum 7, Wieger Wesselink,
- Group 04: Metaforum 8, Jaap van der Woude.

Problem 1.

(20 %) Give an example of a set A and a function $f : A \rightarrow A$ for which $f \circ f$ is surjective but not injective. Motivate your answer.

Problem 2.

(20 %) Consider the poset $(\mathcal{P}(\{1, 2, 3, 4, 5\}), \subseteq)$. Determine all minimal and maximal elements of

$$\{A \subseteq \{1, 2\} \mid \#A \geq 1\} \cup \{A \subseteq \{2, 3, 4, 5\} \mid \#A = 3\}.$$

Problem 3.

(30 %) Let (P, \sqsubseteq) be a lattice. Let $a, b, c, d, e \in P$ satisfy $a \sqsubseteq c$, $b \sqsubseteq c$, $c \sqsubseteq d$ and $c \sqsubseteq e$. Prove that

$$a \sqcup b \sqsubseteq d \sqcap e.$$

Problem 4.

(30 %) Let (P, \sqsubseteq) be a lattice in which $m \in P$ is the maximum of P . For $a \in P$ define $f_a : P \rightarrow P$ by $f_a(x) = x \sqcap a$ for all $x \in P$. Prove that f_a is surjective if and only if $a = m$.