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2ITX0 Applied Logic, Quartile 2, 2021/2022

`https://www.win.tue.nl/~hzantema/al.html`

Teachers: Hans Zantema and Tom Verhoeff

The current course 2ITX0 is mainly for the bachelor Computer Science and Engineering

Students of other programs are welcome too, as long as they are familiar with some basics of programming, and Logic and Set Theory (2IT60 or 2ITS60)

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Logic

Logic deals with **formulas** having a **boolean** meaning: the value of the formula is false or true

In **Logic and Set Theory** we have seen **proposition logic**, using $\vee$, $\wedge$, $\neg$, $\rightarrow$, $\leftrightarrow$, and **predicate logic**, moreover using $\forall$ and $\exists$

In this course we will also see other flavors of logic, in particular extended by **linear inequalities**, and **Hoare logic**: a logic for reasoning on correctness of programs

We start by proposition logic

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SATisfiability

A **propositional formula** is composed from boolean variables and the operators

- $\neg$ (negation, 'not')
- $\vee$ (disjunction, 'or')
- $\wedge$ (conjunction, 'and')
- $\rightarrow$ (implication, $p \rightarrow q \equiv \neg p \vee q$)
- $\leftrightarrow$ (bi-implication, $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$)

This will play a prominent role in this course too

In particular, we will consider SAT(isfiability): given a formula in proposition logic, decide whether you can give values to the variables such that the formula yields true

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SAT/SMT

We will see how several problems can be expressed as SAT problems, and solved by corresponding tools

We will extend the logic by allowing integer or real valued variables, and allow formulas with **linear inequalities**, like $x + 3y - 2z \geq 5$

In this way SAT is extended to **SMT**: Satisfiability Modulo Theories

We will use the tool **Z3** to solve several problems expressed as an SMT problem

The first two weeks = first four lectures will be about SAT/SMT and underlying theory

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Information theory

Logic is about boolean variables, and formulas with boolean result

How many boolean variables are needed to express some information?

The maximum amount of information that can be stored in a single boolean variable is 1 bit

How do you store data, like text, efficiently, that is, in the least number of bits?

Simple method: use a fixed number of bits for every character

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As there are 26 distinct characters, and $26 \leq 32 = 2^5$, 5 bits per character suffices

For including capitals, numbers, and other symbols, ASCII is a standard with 8 bits per character

More efficient: use a lower number of bits for frequent characters, and a higher number of bits for non-frequent characters

In this way frequency /probability comes in

The optimal solution will be presented in this course

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Other issues dealing with information:

- **Coding theory**: what to do when data can be lost?
- **Cryptography**: what to do to keep the data safe / secret?

All these issues concerning information will be presented by Tom Verhoeff in the four lectures in the 3d and 4th week

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Program Correctness

Four lectures in the last two weeks before Christmas

We will present **Hoare logic** to describe that a particular **program** satisfies a particular **specification**

Again the formulas yield false or true, but the formula itself contains the program and its specification

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Program Correctness

We will deal with two ways to solve this kind of problems:

- **Bounded model checking**, for programs doing a fixed number of steps: express in SAT/SMT problem, and let Z3 solve the problem
- Using an **invariant**: a property that holds initially, is maintained in every step, so also holds at the end

  For correct reasoning we give **Hoare logic** in which for all building blocks of programs proof rules are given

  We give examples in which first the invariant is chosen, and then the program is designed in such a way that the invariant properties hold, and the program is correct by construction

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Organization

- Lectures on Wednesday 7+8 and Friday 3+4
- Practical support on Friday 1+2 (no registration required) Here you have the opportunity to work with the tools and to work on the assignments; student assistants will be present to answer questions
- Tutorials on Wednesday 5+6, starting in the second week (register for tutorial groups via Canvas no later than Thursday, November 18)

  Recommended to do the indicated practice set problems in advance; during the tutorial sessions the instructors will explain the solutions of these problems, the problems from the assignments and will answer any new questions that may arise

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Assessment

- Examination: the course is concluded by a written examination that counts for 50 %; the grade for this examination should be at least 5 in order to pass

- Assignments: There are three assignments that should be handed in electronically by every student **individually**, and count for 10 % each

  In the assignments the focus is on doing practical work using tools

  Deadlines: December 3, December 17 and January 14

- Interim tests: On December 3, December 17 and January 14, from 9:00-10:00 there will be interim tests with problems similar to what may be expected for the examination; they count for 5 %, 5 % and 10 %