

TWO-DIMENSIONAL REWRITING TECHNIQUES AND APPLICATIONS

Advanced Track of ISR 2017 - Course Proposal

*Philippe Malbos*¹ and *Samuel Mimram*²

String rewriting systems have originally been introduced in order to provide presentations of monoids in which the word problem can be decided efficiently. Powerful techniques and applications have been developed in this setting: critical pairs, termination orders, completion procedures, etc. *Higher-dimensional rewriting* is motivated by the extension of these tools to richer algebraic settings, and results in an unified and geometrical setting for abstract, string and term rewriting systems, as well as rewriting in higher-dimensional categories. It also provides new tools to study presentations by rewriting of low-dimensional structures such as monoids.

The goal of this course is to provide an introduction to two-dimensional rewriting systems as a setting to study presentations of two-categories, but also to describe confluence properties of string rewriting systems. In a first session, we will study confluence and termination properties of two-dimensional rewriting systems. After defining two-categories and the associated rewriting systems, the notion of critical pair will be discussed in this setting, as well as completion techniques. Several examples will be provided as illustration: associativity, symmetries and commutativity of a binary law. In a second session, we will explain how two-dimensional rewriting systems can still be useful to study monoids, by extending the usual notion of a presentation of a monoid with relations between relations. We will see that it allows one to compute invariants of monoids, will relate those to the decidability of the word problem.

¹Institut Camille Jordan, CNRS UMR 5208, Université de Lyon, malbos@math.univ-lyon1.fr

²LIX, CNRS UMR 7161, École Polytechnique, samuel.mimram@lix.polytechnique.fr