Quantitative Variants of Language Equations and their Applications to Description Logics

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Motivation: Unification in Description Logics

While creating knowledge bases redundancies may occur.

$Woman \sqcap \forall\, has\text{-}child.Woman$
$Human \sqcap Female \sqcap \forall\, has\text{-}child.(Human \sqcap Female)$

Unification in DL can discover and remedy redundancies by introducing unifiers
e.g., $Woman \mapsto Human \sqcap Female$

Problem

Exact unifiers need not always exist
Unification is too restrictive

$Woman \sqcap \forall\, has\text{-}child.Woman$
$Human \sqcap Female \sqcap \forall\, has\text{-}child.Female$

Finding approximate solutions

1. Determine meaningful ways to assess solutions.
2. Provide algorithms for computing optimal solutions.

Approaches: Reduction to Automata Problems

Classical construction (introduced by Baader & Narendran)

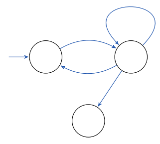
Unification in $\mathcal{FL}_0$ —reduces to→ Language Equations —solved using→ Automata

$C \stackrel{?}{\equiv} D$

$X \sqcap \forall r.X$
$A \sqcap B \sqcap \forall r.(A \sqcap B)$

$K_0 \cup K_1X_1 \cup \cdots \cup K_nX_n$
$= L_0 \cup L_1X_1 \cup \cdots \cup L_nX_n$

$\{\varepsilon, r\}X = \{\varepsilon, r\}$



New construction (JELIA '16)

Approximate Unification in $\mathcal{FL}_0$ —reduces to→ Approximate Language Equations —solved using→ Modified Versions of Automata

Approximate Unification in $\mathcal{FL}_0$ —need→ Measure of Closeness of Concepts

Approximate Language Equations —need→ Distance of Languages

Measure of Closeness of Concepts ←relation?→ Distance of Languages

Modified Versions of Automata —making use of→ Modified Emptiness Test, Linear Programming

Results

Warm-up: Approximate set equations (UNIF '16)

P- or NP-complete, for the measures considered

Language distances induce concept measures

Distances already considered	
$d_1(K, L) = 2^{-n}$ with $n = \min\{\, \lvert w \rvert \;:\; w \in K \Delta L \,\}$	$d_2(K, L) = \mu(K \Delta L)$ with $\mu(L) = \frac{1}{2}\sum_{w \in L}(2\lvert A \rvert)^{-\lvert w \rvert}$

- Existence of optimal solution guaranteed for both cases.
- Approximate unification is ExpTime-complete w.r.t. d_1, d_2.

Current & Future Work

Matching

Special case of unification: $K_0 = L_0 \cup L_1X_1 \cup \cdots \cup L_nX_n$
- The classical case is in P
- Approximate matching w.r.t. d_1 stays in P
  in NP for a wide class of distances

Extensions

- Adding terminological knowledge
  $Human \sqsubseteq \forall\, has\text{-}child.Human$
- Approximate unification and matching in other DLs
- Connection to weighted automata (LATA '17)

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