

Discrete Structures

(6 ects course, last time given in 2011)

Examination 2IT25, November 6, 2012, 14.00 - 17.00

This examination consists of 6 problems each having the same weight.

Solutions may be given in English or Dutch.

Motivate your answers.

Problem 1.

Let R, S be two relations on a set U satisfying $R; S \subseteq S; R$.

- (a) Prove that $R^n; S \subseteq S; R^n$ for all $n \geq 0$.
- (b) Prove that $R^*; S \subseteq S; R^*$.

Problem 2.

Let (V, E) be a finite connected undirected graph. Let W be a finite set and $f : V \rightarrow W$ a bijective function. Prove that the undirected graph

$$(V \cup W, E \cup \{(v, f(v)) \mid v \in V\})$$

is connected.

Problem 3.

A poset (U, \sqsubseteq) is given with two subsets X and Y for which $\sup(X)$ and $\sup(Y)$ exist, and $\sup(X) \in Y$. Prove that $\sup(Y)$ is an upper bound of X .

Problem 4.

Let $(V, *, I, \cdot^{-1})$ be a group for which $a, b \in V$ satisfy

$$a^2 = b^3 = I.$$

Prove that $(a * b^{-1} * a)^3 = I$.

Problem 5.

Find all natural numbers n less than 400 for which $17n - 2$ is divisible by 95.

Problem 6.

Let a_i be defined for $i \geq 0$ by

$$a_0 = a_1 = 2$$

and

$$a_{i+2} = 2(a_{i+1} - a_i)$$

for $i \geq 0$.

Find a closed expression for a_i .