Discrete Structures

(6 ects course, last time given in 2011)

Examination 2IT25, January 24, 2013, 14.00 - 17.00

This examination consists of 5 problems each having the same weight.

Solutions may be given in English or Dutch.

Motivate your answers.

Problem 1.

Let R, S be two relations on a set U.

(a) Prove that $(R;S)^n;R \;=\; R;(S;R)^n$ for all $n \geq 0$.

(b) Prove that $(R;S)^*;R \;=\; R;(S;R)^*$.

Problem 2.

For an undirected graph (V, E) every two non-empty subsets V_1 and V_2 of V satisfy:

$$(V_1 \cup V_2 = V) \Rightarrow (\exists v_1, v_2 : v_1 \in V_1 \wedge v_2 \in V_2 \wedge (v_1, v_2) \in E).$$

Prove that (V, E) is connected.

(Hint: for a node v consider $V_1 = \{u \in V \mid \text{ there is a path from } v \text{ to } u\}$.)

Problem 3.

The sequence a_n satisfies $a_n = 7a_{n-1} - 12a_{n-2}$ for $n \geq 2$. Determine a simple closed form for a_n if it is given that $a_0 = 1$ and $a_1 = 2$.

Problem 4.

Give all solutions of the equation $49x = 21$ in $\mathbb{Z}/105\mathbb{Z}$.

Problem 5.

Let $(G, *, I)$ be a group and let $g \in G$ satisfy $g^n = I$ for some odd number n. Prove that there is an $h \in G$ such that $g = h^2$.