Discrete Structures 2IT50 (5 ects)

Final examination 2IT51, January 30, 2014, 14.00 - 17.00

This examination consists of 5 problems each having the same weight. The final grade is the weighted average of the result of this examination (70 %) and the average of the best two of the three interim tests (30 %).

Solutions may be given in English or Dutch.

Motivate your answers.

Problem 1.

(a) Let R be a transitive relation on a set U. Prove that $R^n \subseteq R$ for all $n \geq 1$.

(b) Let R be an equivalence relation on a set U. Prove that $R^* = R$.

Problem 2.

(a) Show that for every $n \geq 3$ a tree exists with exactly n nodes and $n-1$ leaves.

(b) Prove that every connected undirected graph with 10 nodes and 10 edges contains a cycle.

Problem 3.

(a) In the poset $(\mathcal{P}(\{1,2,3,4\}, \subseteq)$ give all minimal and maximal elements of $\{\{1,2\},\{1,2,3\},\{1,3,4\},\{1,2,4\}\}$.

(b) Let $(U, \sqsubseteq)$ be a poset, and let $A, B, C \subseteq U$ such that $A \subseteq B \subseteq C$, $\sup(A)$ and $\sup(C)$ exist, and $\sup(A) = \sup(C)$. Prove that $\sup(B)$ exists and $\sup(B) = \sup(A)$.

Problem 4.

Let $(G, *, I)$ be a group and let $a \in G$. Let $f : G \to G$ be the function defined by $f(x) = x * a$ for all $x \in G$. Prove that f is bijective.

Problem 5.

(a) Find a closed expression for a_i defined by $a_0 = 2$, $a_1 = 1$, $a_i = a_{i-1} + 6a_{i-2}$ for $i > 1$.

(b) Compute the prime factorization of the least common multiple of 7! and $\binom{11}{3}$.