Discrete Structures 2IT50

Final examination 2IT51, November 6, 2014, 9.00 - 12.00

This examination consists of 7 problems each having the indicated weight. The final grade is the weighted average of the result of this examination (70 %) and the average of the best two of the three interim tests (30 %).

Solutions may be given in English or Dutch.

Motivate your answers.

Problem 1.

(a) (15 %) Let R be a transitive relation on a set U. Prove that

$$R^{n+1} \subseteq R^n$$

for all $n \geq 1$.

(b) (5 %) Give an example of a relation R on a finite set such that $R^3 \not\subseteq R^2$.

Problem 2.

(15 %) Let (V, E) be an undirected acyclic graph having exactly two connected components. Prove that $\#E = \#V - 2$.

Problem 3.

(a) (5 %) Explain what is meant by topological sorting.

(b) (15 %) Let $(U, \sqsubseteq)$ be a poset, and let $A \subseteq U$ have both a supremum s and a maximum m. Prove that $s = m$.

Problem 4.

(15 %) Let $(G, *, I)$ be a group and let $a, b \in G$. Prove that $a^3 = I$ if and only if $(b^{-1} * a * b)^3 = I$.

Problem 5.

(10 %) Find a closed expression for a_i defined by $a_0 = a_1 = 1$, and

$$a_i = 2a_{i-1} + 3a_{i-2}$$

for $i > 1$.

Problem 6.

(10 %) Apply the extended Euclidean algorithm to find two integer numbers x, y such that $21x + 8y = 1$.

Problem 7.

(10 %) Compute the greatest common divisor of $8!$ and $\binom{12}{5}$.