Discrete Structures 2IT50

Final examination 2IT51, January 29, 2015, 18.00 - 21.00

This examination consists of 8 problems each having the indicated weight. The final grade is the weighted average of the result of this examination (70 %) and the average of the best two of the three interim tests (30 %).

Solutions may be given in English or Dutch.

Motivate your answers.

Problem 1.

(10 %) Give an example of a transitive relation that is not anti-symmetric.

Problem 2.

(15 %) Let R, S be relations on a set U of which R is transitive. Prove that

$$(R;S;R)^n \subseteq (R;S)^n;R$$

for all $n \geq 1$.

Problem 3.

(15 %) Given is an undirected graph (V, E) in which

$$2\#E > \#V.$$

Prove that in this graph there is path uvw for which $u \neq w$.

Problem 4.

(15 %) Let $(U, \sqsubseteq)$ be a poset, and let $A, B \subseteq U$ both have a supremum and an infimum, and satisfy $a \sqsubseteq b$ for all $a \in A$ and all $b \in B$. Prove that $\sup(A) \sqsubseteq \inf(B)$.

Problem 5.

(10 %) Give an example of an undirected graph that has a Hamiltonian cycle but does not have an Euler cycle.

Problem 6.

(15 %) Let $(G, *, I)$ be a group and let $a \in G$. Define $f : G \to G$ by

$$f(x) = (a * x)^{-1}$$

for all $x \in G$. Prove that f is bijective.

Problem 7.

(10 %) The sequence $a_0, a_1, a_2, a_3, \ldots$ satisfies $a_0 = \frac{3}{2}$, $a_3 = 31$, and

$$a_i = 3a_{i-1} + 4a_{i-2}$$

for all $i > 1$. Note that a_0, a_3 are given and not a_1. Find a closed expression for $a_0, a_1, a_2, a_3, \ldots$.

Problem 8.

(10 %) Give the prime factorization of the least common multiple of $7!$ and $\binom{13}{4}$.