Discrete Structures 2IT50

Final examination 2IT51, November 5, 2015, 9.00 - 12.00

This examination consists of 8 problems each having the indicated weight. The final grade is the weighted average of the result of this examination (70 %) and the average of the best two of the three interim tests (30 %).

Solutions may be given in English or Dutch.

Motivate your answers.

Problem 1.

(10 %) Give an example of a relation R on a finite set such that $R^* \neq (R;R)^*$.

Problem 2.

(15 %) Let R be a transitive relation on a set U. Prove that

$$R^{2n} \subseteq R^{n+1}$$

for all $n \geq 1$.

Problem 3.

(15 %) Let two non-empty sets A, B satisfy $A \cap B = \emptyset$. For a function $f : A \to B$ define the undirected graph $G_f = (V, E)$ for which $V = A \cup B$ and

$$E = A \times A \cup \{(x, f(x)) \mid x \in A\}.$$

Prove that G_f is connected if and only if f is surjective.

Problem 4.

(15 %) Consider the poset $(\mathbb{N}, |)$ and let $n > 0$. Define

$$A = \{x \in \mathbb{N} \mid n \leq x \leq 2n\}.$$

Determine all minimal elements of A and prove that there are exactly n of them.

Problem 5.

(15 %) Let $(G, *, I)$ be a group and let $a, b \in G$ such that $a * b$ has order 2. Prove that $b * a$ has order 2.

Problem 6.

(10 %) Find a closed expression for a_n defined by $a_0 = a_1 = 1$, and

$$a_n = 2a_{n-1} + 15a_{n-2}$$

for $n \geq 2$.

Problem 7.

(10 %) Apply the extended Euclidean algorithm to find an integer number x such that

$$(17x \textbf{ mod } 83) = 1.$$

Problem 8.

(10 %) Let $n = 2k + 1 > 10$ and $(2^k \textbf{ mod } n) = 3$. Prove that n is not a prime number.
(Hint: note that $2^{n-1} = (2^k)^2$)