

# Discrete Structures 2IT50

Final examination 2IT51, January 28, 2016, 18.00 - 21.00

This examination consists of 8 problems each having the indicated weight. The final grade is the weighted average of the result of this examination (70 %) and the average of the best two of the three interim tests (30 %).

Solutions may be given in English or Dutch.

Motivate your answers.

## Problem 1.

(10 %) Give an example of a relation  $R$  on a finite set that is reflexive and symmetric, but not an equivalence relation.

## Problem 2.

(15 %) The relation  $R$  from  $U$  to  $V$  is an injective function. Prove that  $R; R^T = I_U$ .

## Problem 3.

(15 %) Let  $(V, E)$  be an undirected tree. Let  $v \in V$  and  $w \notin V$ . Prove that  $(V \cup \{w\}, E \cup \{(v, w)\})$  is a tree.

## Problem 4.

(10 %) Let  $m \neq m'$  be two minimal elements of a set  $A \subseteq U$  in a poset  $(U, \sqsubseteq)$ . Prove that  $A$  has no minimum.

## Problem 5.

(15 %) Let  $(U, \sqsubseteq)$  be a lattice and  $a, b, c \in U$ . Prove that

$$(a \sqcap b) \sqcup c \sqsubseteq b \sqcup (a \sqcup c).$$

## Problem 6.

(15 %) Let  $(G, *, I)$  be a group. Let  $x, y \in G$  both have finite order. Prove that there exist  $m, n \geq 0$  such that  $(x * y)^{-1} = y^m * x^n$ .

## Problem 7.

(10 %) Find a closed expression for  $a_n$  defined by  $a_0 = 0$ ,  $a_1 = 1$ , and

$$a_n = 4a_{n-1} - 2a_{n-2}$$

for  $n \geq 2$ .

## Problem 8.

(10 %) Compute the greatest common divisor of  $10!$  and  $\binom{11}{4}$ .