Name:

Student number: | | | | | | | |

1	2	3	4	5	6	7	Σ

Final exam 2IT51 Discrete Structures 2IT50, November 10, 2016

This examination consists of 7 problems with the indicated weights.

In giving proofs you may use theorems and lemmas from the lecture notes (not exercises), as long as you indicate that you use them.

The problems may be solved either in English or in Dutch. Please write your final answer on this paper in the indicated boxes (after preparing on scrap paper). If it does not fit, you may hand in an extra sheet.

Problem 1.

(10 %) Give an example of an irreflexive relation R on $U = \{0, 1, 2, 3\}$ such that R has exactly four elements and $(U, R \cup I_U)$ is a poset. Draw the corresponding Hasse diagram.

Problem 2.

Let R, S be relations on a set U satisfying $R; S \subseteq S^3$.

a. (15 %) Prove by induction that $R^n; S \subseteq S^{2n+1}$ for all $n \geq 0$.

b. (10 %) Prove that $R^*; S \subseteq S^+$.

Problem 3.

Let $T = (V, E)$ be a finite tree, and $v, w \in V$ satisfy $v \neq w$, $(v, w) \notin E$. Let $E' = E \cup \{(v, w)\}$ and $G = (V, E')$.

a. (10 %) Prove that G contains a cycle.

b. (10 %) Let (x, y) be any edge on a cycle in G. Prove that $H = (V, E' \setminus \{(x, y)\})$ is a tree.

Problem 4.

(15 %) Let $(U, \sqsubseteq)$ be a poset, and let $A, B, C \subseteq U$ such that $A \subseteq B \subseteq C$, $\sup(A)$ and $\sup(C)$ exist, and $\sup(A) = \sup(C)$.

Prove that $\sup(B)$ exists and $\sup(B) = \sup(A)$.

Problem 5.

(10 %) Let $(M, *, I)$ be a monoid and let $a, b \in M$ satisfy $a * b * b = b * a$ and $b^7 = I$. Prove that $(a * b)^3 = a^3$.

Problem 6.

(10 %) How many sequences $a_1, a_2, a_3, a_4, a_5, a_6$ exist for which $a_i \in \{1, 2, 4, 8\}$ for $i = 1, \ldots, 6$ and $a_i \geq a_{i+1}$ for $i = 1, \ldots, 5$? (Only the answer as a number is sufficient)

Problem 7.

(10 %) Use the extended Euclidean algorithm to find a positive integer n such that that $17n - 1$ is divisible by 19.