Name:

Student number: | | | | | | | |

1	2	3	4	5	6	7	8	Σ

Final exam 2IT51 Discrete Structures 2IT50, February 2, 2017

This examination consists of 8 problems with the indicated weights.

In giving proofs you may use theorems and lemmas from the lecture notes (not exercises), as long as you indicate that you use them.

The problems may be solved either in English or in Dutch. Please write your final answer on this paper in the indicated boxes (after preparing on scrap paper). If it does not fit, you may hand in an extra sheet.

Problem 1.

(10 %) Give an example of an equivalence relation R on $U = \{0, 1, 2, 3, 4, 5\}$ such that R has exactly three equivalence classes, all having distinct sizes.

Problem 2.

Let R, S be relations on a set U satisfying $R; S \subseteq S; R$.

a. (10 %) Prove by induction that $R^n; S \subseteq S; R^n$ for all $n \geq 0$.

b. (10 %) Prove by induction that $(R; S)^n \subseteq S^n; R^n$ for all $n \geq 0$.

Problem 3.

(15 %) Let $G = (V, E)$ be a finite connected undirected graph and let $f : W \rightarrow V$ for a finite set W, $V \cap W = \emptyset$. Prove that $(V \cup W, E \cup \{(w, f(w)) \mid w \in W\})$ is connected.

Problem 4.

(10 %) Give an example of a poset $(U, \sqsubseteq)$ and $A, B \subseteq U$ such that $\sup(A)$ and $\sup(B)$ exist, and $s = \sup(A) = \sup(B)$, and $s \in A$ and $s \notin B$.

Problem 5.

(10 %) Compute the greatest common divisor of 15! and $\binom{17}{3}$. (only the answer as a number is sufficient)

Problem 6.

(15 %) Let $(U, \sqsubseteq)$ be a total poset and let $a, b \in A \subseteq U$ such that both a and b are minimal in A. Prove that $a = b$ and a is the minimum of A.

Problem 7.

(10 %) Let $(G, *, I)$ be a group and let $a, b \in G$ satisfy $a^{-1} * b = (a * b^{-1})^{-1}$. Prove that $a * b = b * a$.

Problem 8.

(10 %) Solve the following recurrence relation:

$$a_0 = a_1 = 1; a_{n+2} = a_{n+1} + 20a_n \text{ for } n \geq 0.$$