Final exam Discrete Structures 2IT50,
November 9, 2017, 9:00 - 12:00

Name:

Student number: | | | | | | | |

1	2	3	4	5	6	7	Σ

This examination consists of 7 problems with the indicated weights.

In giving proofs you may use theorems and lemmas from the lecture notes (not exercises), as long as you indicate that you use them.

Please write your final answer on this paper in the indicated boxes (after preparing on scrap paper). If it does not fit, please indicate and hand in an extra sheet.

Problem 1.

(10 %) Give an example of a relation R on $U = \{1, 2, 3\}$ such that R has exactly three elements, and R is not anti-symmetric, and R is not transitive.

Problem 2.

Let R, S be relations on a set U satisfying $R; S \subseteq S; S; R$.

a. (15 %) Prove by induction that $R; S^n \subseteq S^{2n}; R$ for all $n \geq 0$.

b. (10 %) Prove that $R; S^+ \subseteq S; S^+; R$.

Problem 3.

(15 %) Let $G = (V, E)$ be a finite connected undirected graph, W a set satisfying $V \cap W = \emptyset$, and $f : V \to W$ a surjective function. Prove that the graph

$$G' = (V \cup W, E \cup \{(v, f(v)) \mid v \in V\})$$

is connected.

Problem 4.

(15 %) Let $(U, \sqsubseteq)$ be a lattice, and let $x, y, z \in U$. Prove that $x \sqsubseteq (x \sqcup y) \sqcap (z \sqcup x)$.

Problem 5.

(15 %) Let $(G, *, I)$ be a group in which every $x \in G \setminus \{I\}$ has order 2. Prove that G is abelian.

Problem 6.

(10 %) Let A be set of 30 elements. Compute $gcd(m, n)$, where m is the number of sequences of length 5 over A, and n is the number of subsets of A of exactly 5 elements. (Only the answer is sufficient)

Problem 7.

(10 %) Find a closed expression for a_i defined by $a_0 = a_1 = 1$, and

$$a_i = 2a_{i-1} + 3a_{i-2}$$

for $i > 1$.