

**Final exam Discrete Structures 2IT50,
February 1, 2018, 18:00 - 21:00**

Name:

Student number:

1	2	3	4	5	6	7	Σ

This examination consists of 7 problems with the indicated weights.

In giving proofs you may use theorems and lemmas from the lecture notes (not exercises), as long as you indicate that you use them.

Please write your final answer on this paper in the indicated boxes (after preparing on scrap paper). If it does not fit, please indicate and hand in an extra sheet.

Problem 1.

(10 %) Give an example of a set A and a function $f : A \rightarrow A$ such that $f \circ f$ is injective but not surjective. Motivate your answer.

Problem 2.

- a. (15 %) Let R be a transitive relation on a set U . Prove by induction that $R^n \subseteq R$ for all $n \geq 1$.
- b. (10 %) Let R be an equivalence relation on a set U . Prove that $R^* = R$.

Problem 3.

(15 %) Let (V, E) be an undirected acyclic graph having exactly two connected components. Prove that $\#E = \#V - 2$.

Problem 4.

(15 %) Let (U, \sqsubseteq) be a poset, and let $A \subseteq U$ have an element m that is both a minimum and a maximum of A . Prove that $\#A = 1$.

Problem 5.

(15 %) Let $(M, *, I)$ be a monoid in which $x^k = I$ for some $x \in M$ and some odd number $k > 1$. Prove that there exists $y \in M$ such that $x = y * y$.

Problem 6.

(10 %) Find an integer k for which $0 < k < 103$ and $51k - 3$ is divisible by 103. (Only the answer is sufficient)

Problem 7.

(10 %) Compute the prime factorization of the least common multiple of $8!$ and $\binom{13}{3}$.