

# Health Care Management and Modelling

Ivo Adan



## Tools

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- Main site for simulation language  $\chi$ 3.0 is <http://chi.se.wtb.tue.nl> including:
  - Tool manual (installation, use of software)
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- Software package **R** software is recommended for statistical computing (distribution plots, histograms, ...)

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- Able to construct and analyze elementary queueing models
- Getting hands-on experience with simulation language  $\chi$ 3.0
- Develop intuition and understanding of critical logistical parameters
- Develop understanding of the power and limitations of stochastic models for health care systems

## Modeling: basic steps

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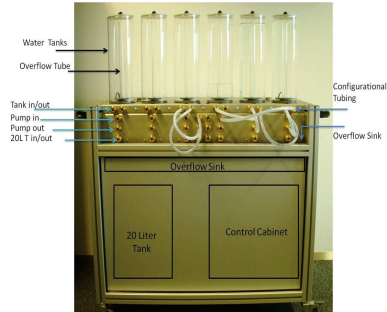
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- Present the results!

## Modeling: Types of models

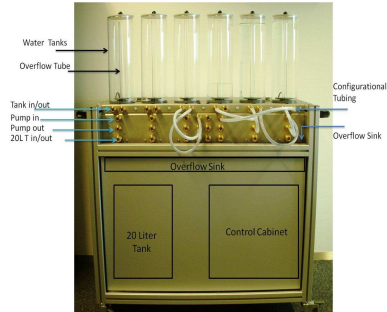
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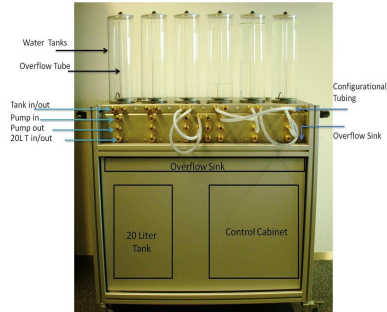
- Simulation models ( $\chi$  3.0 code)

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model result GRSE():
  chan patient a, b, c;
  run G(a, exponential(6.0)),
      R(a, b),
      S(b, c, gamma(4.0,1.0)),
      E(c, 6000.0)
end
  
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- Analytical models (Queueing formulas)

$$E(W) = \frac{1}{2} (1 + c_b^2) \frac{\rho}{1 - \rho} E(B)$$

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- Effective modeling requires intuition, analytical and simulation capability
- **Art of modeling** is in the selection of the right model for a given situation

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- **Utilization:** Fraction of time resource (bed, room, nurse, ...) is being used

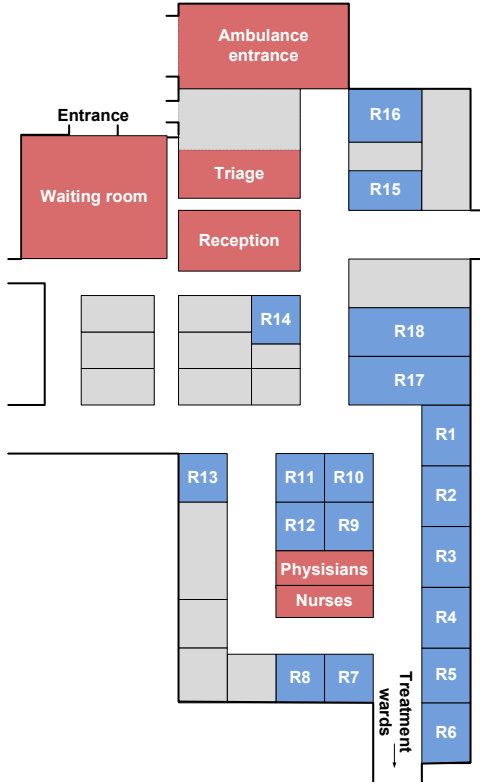


## Emergency Department



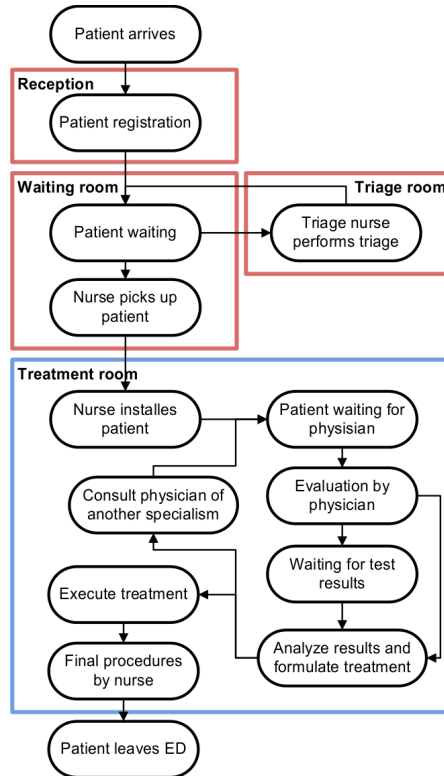
Emergency department (ED) of Catherina Hospital Eindhoven

# Emergency Department



Layout of ED

## Emergency Department



Patient flow through ED

## Goal

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- Develop model to **support decision making** in LEAN process improvement programs
- Address questions such as:
  - What capacity is required to meet target **maximal waiting times**?
  - How much does waiting time decrease by increasing nursing staff?
  - What is the effect of an increase in inflow due to the aging population?

## Queueing model: Basic elements

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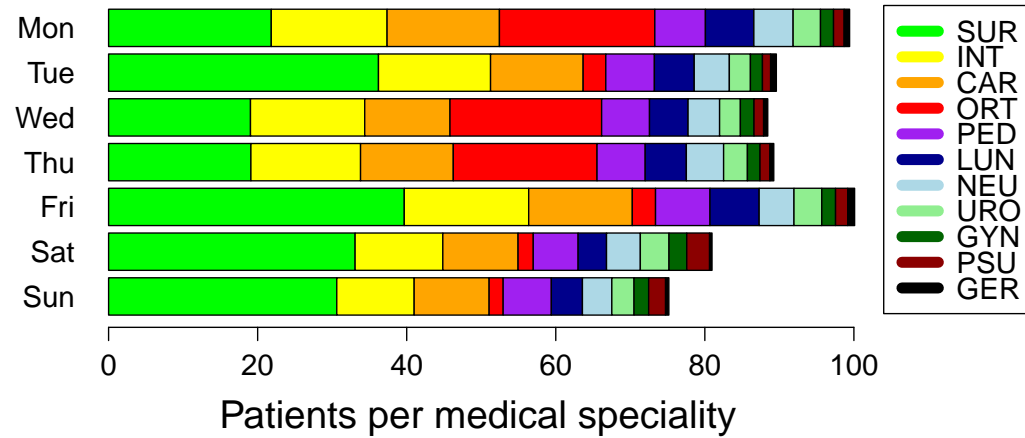
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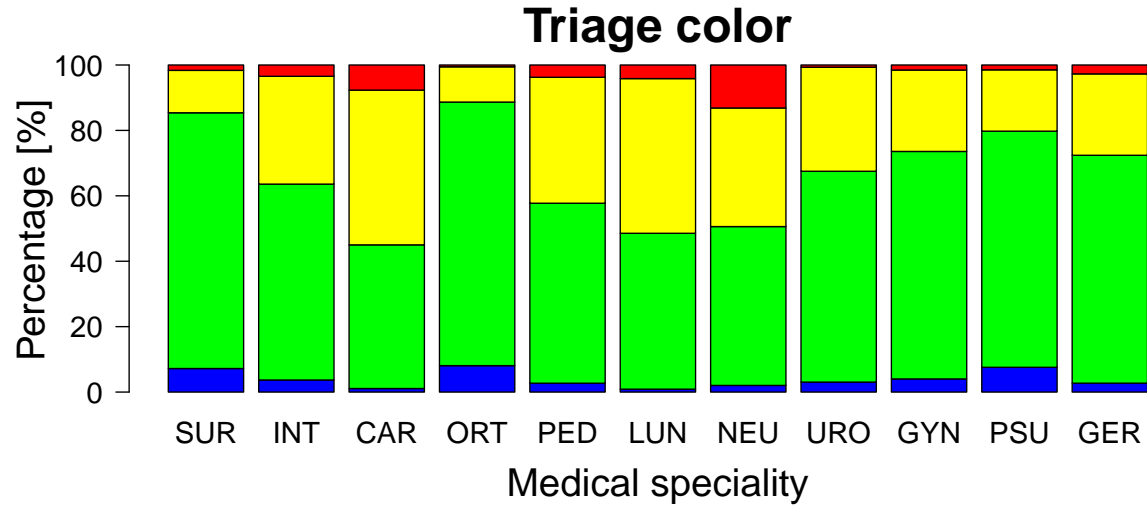
- Patient arrivals
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- Resource capacities

## Arrivals and diversity

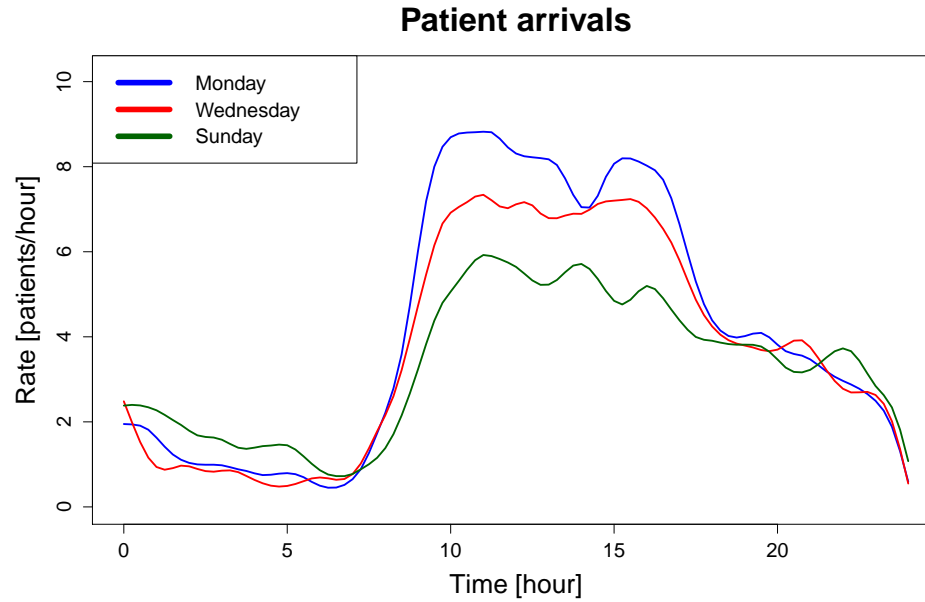
### Distribution ED patients on speciality



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$$r_a = \frac{1}{t_a}$$

- Coefficient of variation of time between arrivals

$$c_a = \frac{\sigma_a}{t_a}$$



Low  $c_a$  arrivals



High  $c_a$  arrivals

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$$t_a = \frac{1}{\lambda}, \quad r_a = \lambda, \quad c_a = 1$$

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- Memoryless property

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So in each small interval  $\Delta$  there is an arrival with probability  $\lambda\Delta$ !

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- This means: “truly unpredictable arrivals”



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- Dividing  $(0, t)$  into intervals of length  $\Delta$ , the number of arrivals in  $(0, t)$  is **Binomial** with  $n = \frac{t}{\Delta}$  and  $p = \lambda\Delta$
- Since  $n$  is large and  $p$  is small, this number is **Poisson distributed** with parameter  $np = \lambda t$

$$P(k \text{ arrivals in } (0, t)) = e^{-\lambda t} \frac{(\lambda t)^k}{k!}, \quad k = 0, 1, 2, \dots$$

This explains the name “**Poisson process**”

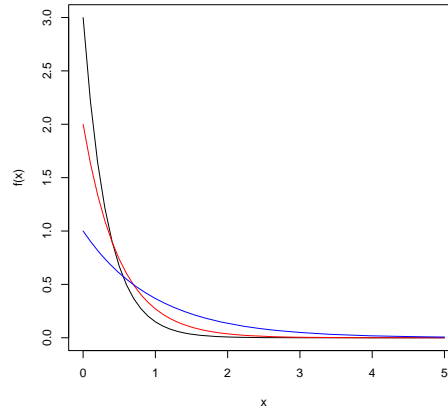
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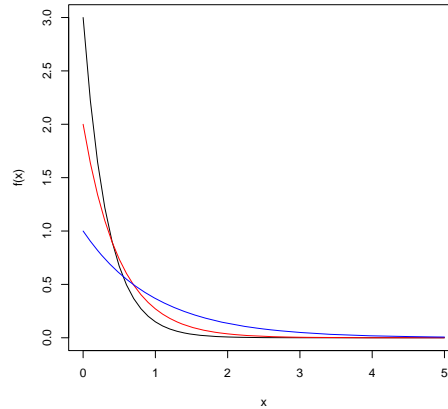


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- So arrivals tend to **cluster**:



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- Superposition of many independent rarely occurring arrival flows is (close to) Poisson
- This explains why Poisson flows so often occur in practice!

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- **Merging of two Poisson flows** with rates  $\lambda_1$  and  $\lambda_2$  is again Poisson with rate  $\lambda_1 + \lambda_2$ , since

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Given there is an arrival in  $(t, t + \Delta)$ , it is of type 1 with probability

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- **Random splitting of Poisson flows** with rate  $\lambda$  and splitting probability  $p$  is again Poisson with rate  $p\lambda$ , since

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- **Answer:** Suppose there is a maximum rate  $\Lambda = \max_{t \geq 0} \lambda(t)$ 
  - Simulate Poisson flow with **constant** rate  $\Lambda$
  - When arrival at time  $t$ , then:
    - ▶ **accept** arrival with probability  $\frac{\lambda(t)}{\Lambda}$
    - ▶ **reject** arrival otherwise

## Treatment times

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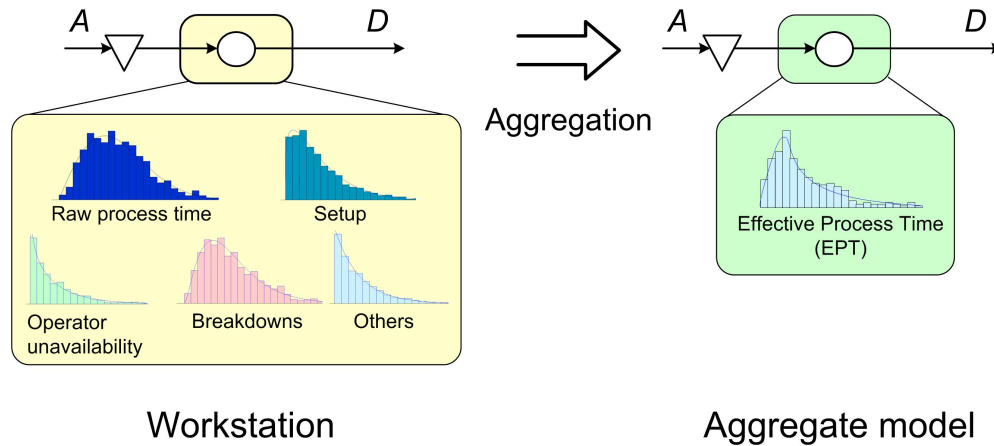
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- Employ the concept of **Effective Process Times**

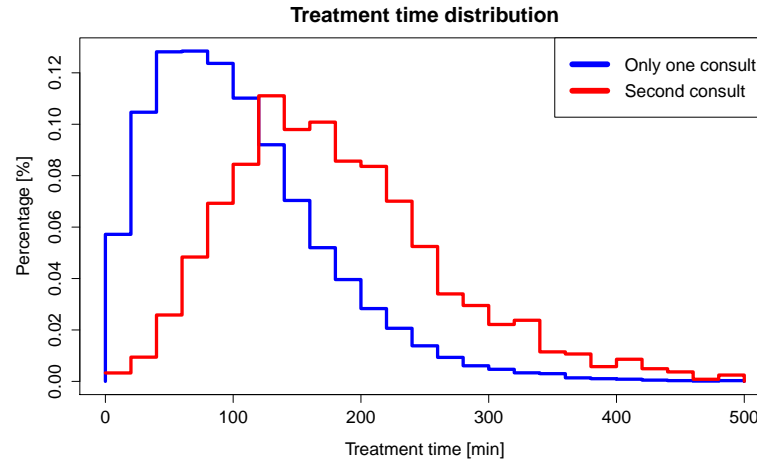


## Treatment times



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- Treatment times of patients depend on patient characteristics such as:
  - Medical speciality
  - Triage color
  - Age
  - Type of attending physician



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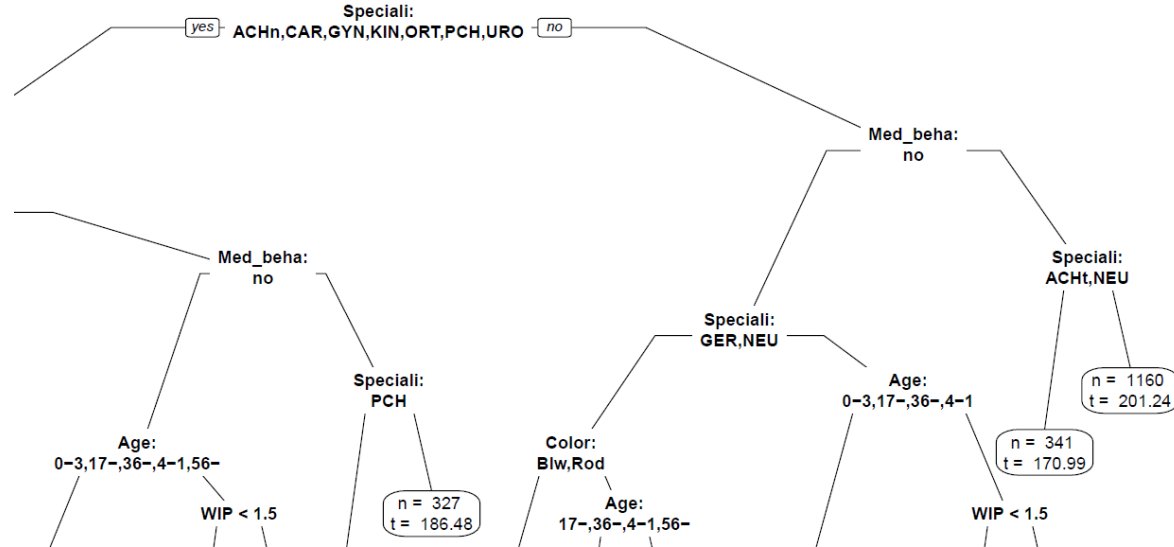
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- **Recursive partitioning** leads to 34 groups of treatment times on which a distribution can be **reliably fitted**



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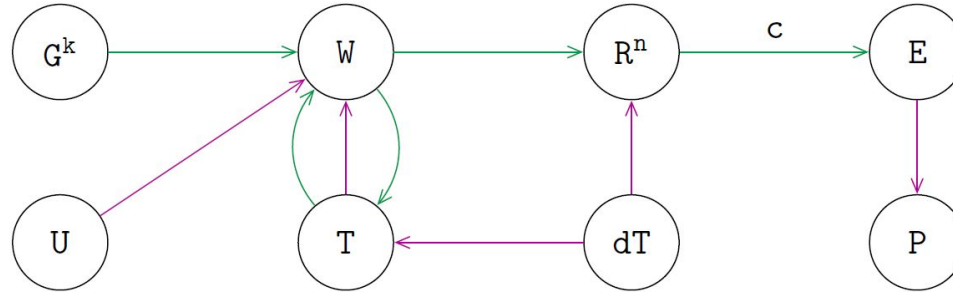
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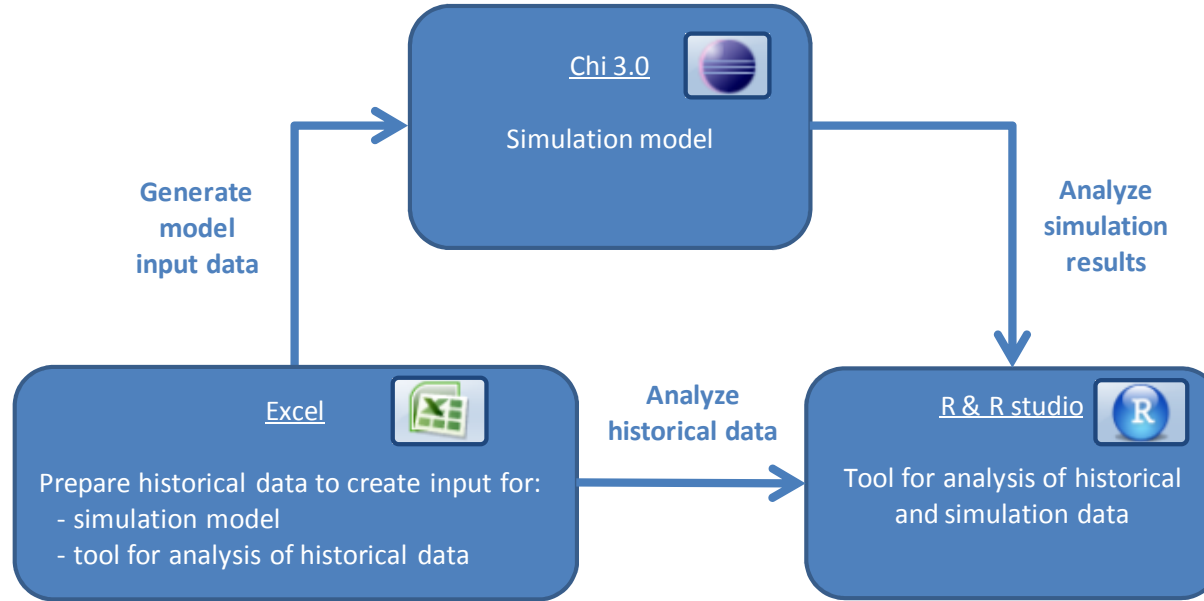
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- Staffing levels adapted to workload during the day:  
Working rosters for each weekday specifying the available capacity at each point in time during the day

## Simulation model

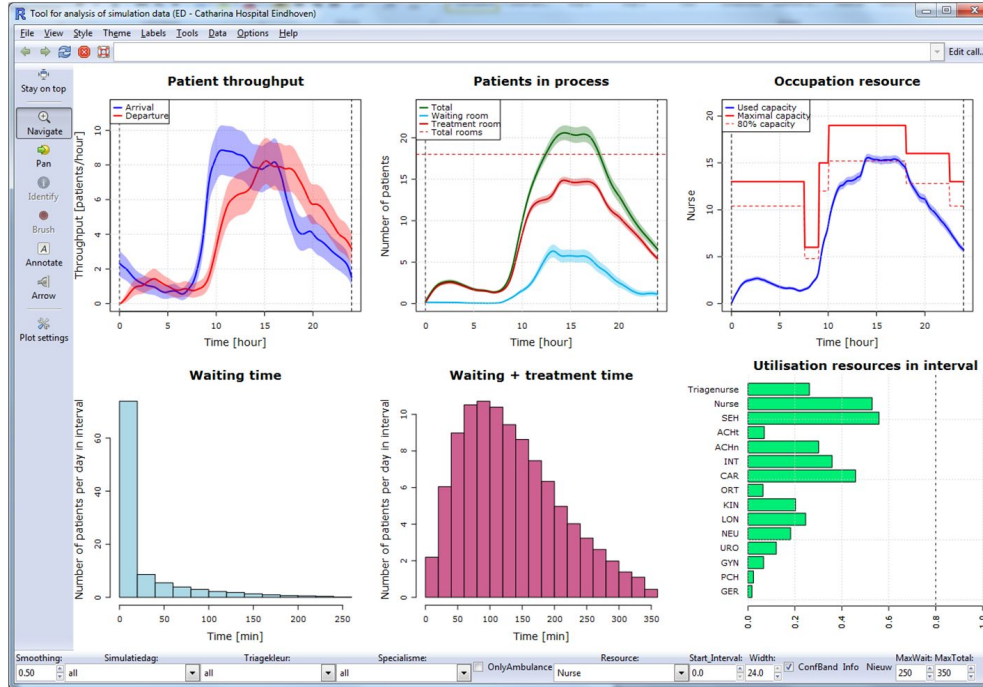


High level  $\chi 3.0$  model of ED: Green is patient flow, Purple information flow

## Software package



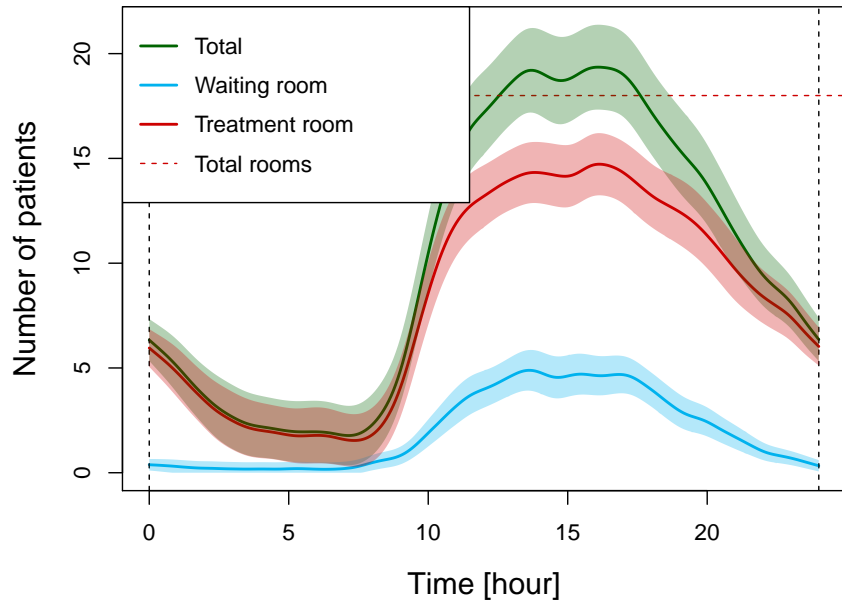
# Emergency Department



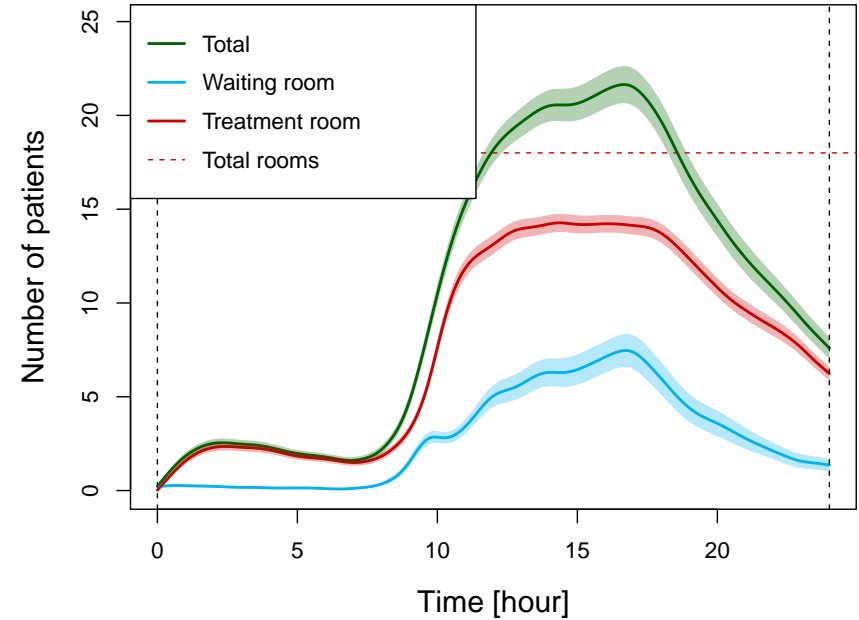
Snapshot of simulation output of  $\chi$ 3.0 model

## Validation

### Patients in process



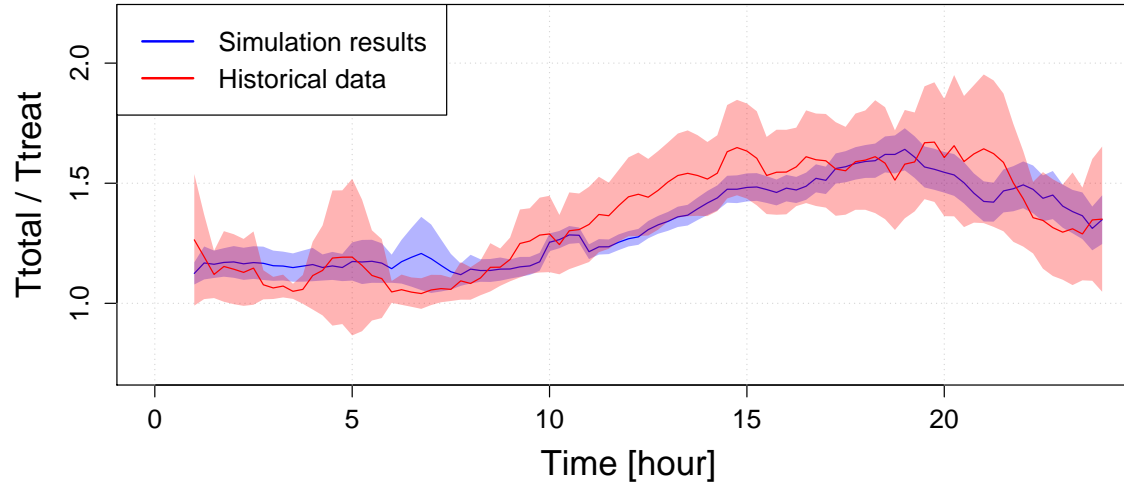
### Patients in process



Historical (left) and simulated (right) average occupation on Monday

## Validation

### Cycle time factor



Cycle time factor for historical and simulated patients on Monday

## Decision support

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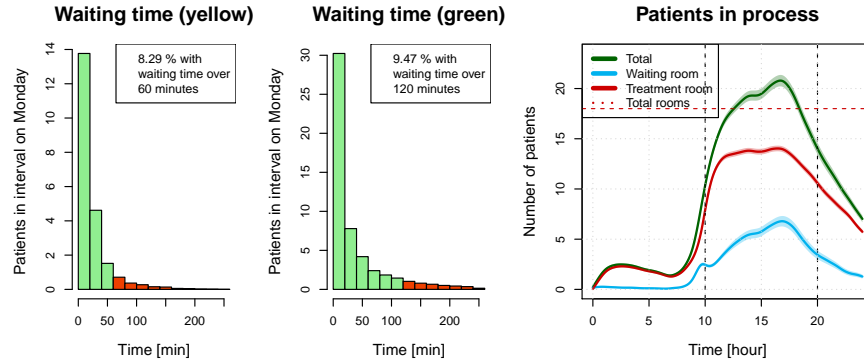
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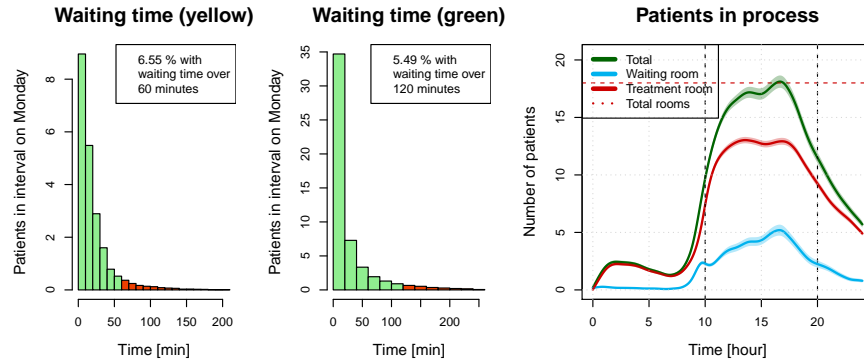
### Options:

- More treatment rooms
- No priority for ambulance patients
- More nursing capacity
- More physician capacity
- Treatment time reduction (by shortening time to hospitalization)

## Treatment time reduction (10 mins)



## Simulation output on Monday for unadapted treatment time



## Simulation output for 10 minutes treatment time reduction



## Scenario analysis

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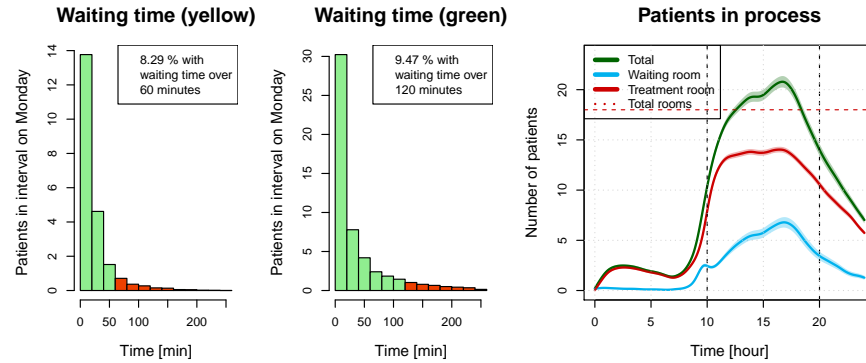
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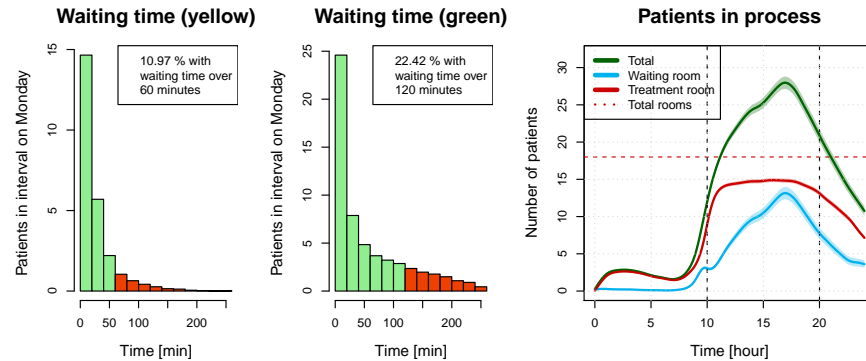
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- What effect has an increase of ED visits by elderly patients?
- What extra capacity is needed if neighboring ED closes?
- What if average urgency of patients increases (due to less self-referrals)?
- What is more accurate triage results in less second consults?

## Scenario: growth arrival rate (15%)



## Simulation output on Monday for unadapted arrival rate



## Simulation output for 15% growth of patient arrivals

## References

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- Aggregate model based performance analysis of an emergency department



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